Coupling Trapped Ions to a Nanomechanical Oscillator

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Cold ions in traps are well-established, highly controllable systems with a wide variety of applications in quantum science, precision spectroscopy, clocks, and chemistry. Nanomechanical oscillators are used in advanced sensing applications and for exploring the border between classical and quantum physics. Here, we report on the implementation of a hybrid system combining a metallic nanowire with laser-cooled ions in a miniaturized ion trap. Operating the experiment in the classical regime, we demonstrate resonant and off-resonant coupling of the two systems and the coherent motional excitation of the ion by the mechanical drive of the nanowire. The present results prove the feasibility of mechanically coupling ions to nanooscillators and open up avenues for mechanically manipulating the motion of trapped ions as well as for the development of ion-mechanical hybrid quantum systems.

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Laser-cooled ions stored in harmonic traps [1] count among the best controlled physical systems [2]. They are a leading platform for quantum computing [3–5] and quantum simulation [6–8], form the basis of some of today's most accurate clocks [9], and have enabled new types of ion-neutral collision and chemical experiments at very low energies [10–12]. Their unique properties have also been harnessed in hybrid experiments with ultracold atoms [13,14].

Traditionally, trapped ions have been manipulated using laser [2], electric [15,16], magnetic, and microwave [17,18] fields. An intriguing prospect is their manipulation by other types of oscillators, e.g., other trapped ions [19–21] or nanomechanical systems. As explored theoretically in Refs. [22–27], ion-mechanical hybrid systems offer possibilities for extending the available techniques for the cooling, manipulation, control, and readout of both constituents with the ultimate aim of realizing novel hybrid quantum systems [28,29] in which entanglement between ions and mechanical objects can be realized.

Similarily, the research on mechanical oscillators on the nanometer scale has progressed rapidly in recent years. Their properties as objects on the border between classical and quantum physics make them intriguing candidates for the realization of ion-mechanical hybrid systems [30]. Cryogenic cooling of nanomechanical oscillators in combination with other cooling techniques has made it possible to prepare them [31–35]. They have also been successfully coupled to ultracold atoms [36–40].

Here, we present an experiment combining trapped, laser-cooled $^{40}Ca^{+}$ ions in a miniaturized linear radio frequency (RF) ion trap with a conductive nanowire [41] as previously explored theoretically in Ref. [27]. A bias voltage applied to the nanowire coupled the two systems by their mutual electrostatic interaction. Operating in the classical regime, the present setup represents a first step towards realizing a quantum hybrid system. We realized the resonant and off-resonant coupling of the two systems and the coherent motional excitation of the ion by the mechanical drive of the nanowire. Resonant excitation of the motion of trapped ions, ranging from single ions to small linear Coulomb crystals, was achieved and manipulation of the ion motion was demonstrated by the variation of different coupling parameters.

Figure 1 shows a schematic of the experimental setup with the most important components. Ions were confined

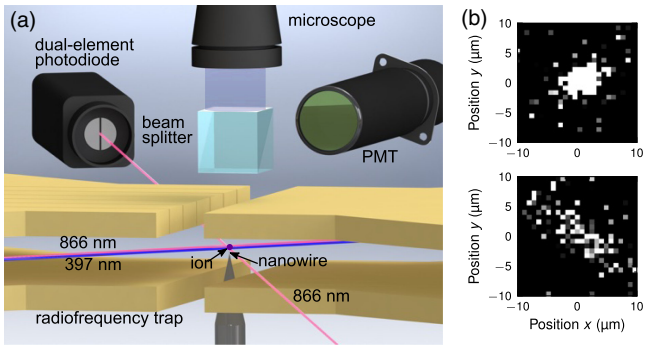

FIG. 1. (a) Schematic of the experimental setup, see text for details. (b) Fluorescence image of a single trapped ion at rest (top) and driven by the nanowire (bottom).

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and laser cooled in a microstructured linear-quadrupole RF trap [10]. The trap consisted of four wafers at a separation of 400 μm fabricated from laser-cut aluminium oxide sputtered with gold following the layout proposed in Ref. [27]. Two diagonally opposed wafers formed the RF electrodes (frequency $f_{\mathrm{RF}} = 21.629$ MHz, $V_{\mathrm{RF}} \approx 50$ V). The remaining two wafers were sectioned into seven individually addressable segments of 400 μm length to which static voltages were applied for confining the ions along the longitudinal axis of the trap and manipulating the position of the ions. The secular oscillation frequencies of the ions were measured using resonant motional excitation by additional RF fields applied to one of the trap electrodes [42]. $^{40}Ca^{+}$ ions were loaded by photoionization of Ca atoms emanating from an oven source using two diode laser beams at 423 nm and 375 nm inside the trap. The ions were Doppler laser cooled on the $(4s)^2S_{1/2} \leftrightarrow (4p)^2P_{1/2} \leftrightarrow (3d)^2D_{3/2}$ system of optical cycling transitions using diode laser beams at 397 nm and 866 nm [10]. The laser beams were inserted at an angle of ≈45° to all principal trap axes (see Fig. 1) to ensure cooling in all spatial directions. The ions were observed by imaging their resonance fluorescence generated during laser cooling onto an electron-multiplying charge-coupled device camera coupled to a microscope [Fig. 1(b)].

The nanomechanical oscillator used was a Ag_2Ga cantilever positioned at the tip of a tungsten support with 250 μm base diameter and ≈4.8 mm length. The length of the nanowire was measured with a microscope to be ≈16 μm with a diameter of ≈150 nm as specified by the manufacturer. The oscillator was mounted on three nanopositioners allowing its free deployment in all spatial directions inside the trap. The wire was mechanically driven in the direction of the longitudinal trap axis (z) by a piezoelectric actuator attached to the assembly.

With the ions cooled close to the Doppler temperature and the nanowire kept at room temperature, the number of motional quanta of the ion and nanowire were estimated to be of order 50 and 10^7, respectively. Under these conditions, both subsystems behaved as coupled classical oscillators.

The eigenmodes and, therefore, also the vibration frequencies of mechanical oscillators are determined by their mass, material, and geometry. To characterize the vibrational modes of the present nanowire, the drive frequency f_{drive} applied to the piezo was scanned. Its oscillations were measured by imaging the tip of the oscillator placed in the waist of a focused laser beam onto a dual-element Si PIN photodiode [43,44]. Figure 2(a) shows a mechanical excitation spectrum thus obtained in the frequency interval from 100 to 500 kHz.

In this range, the strongest mechanical response was found around $f_{\mathrm{drive}} = 422$ kHz. Structural mechanics simulations performed with the COMSOL Multiphysics program [45] indicate that this resonance corresponds to the $n = 4$ eigenmode [46] which represents a combined

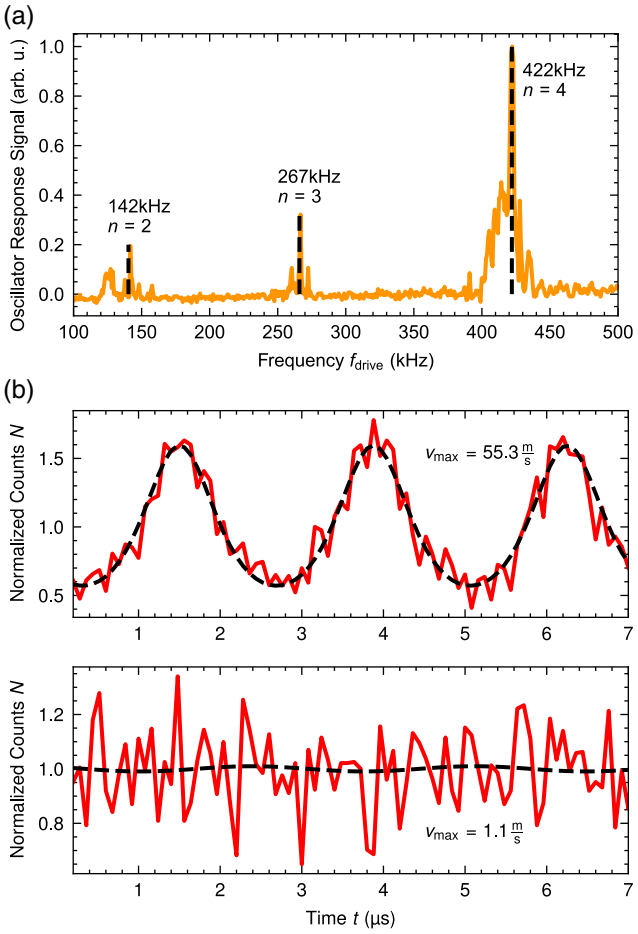

FIG. 2. (a) Mechanical excitation spectrum of the nanowire. The features represent excitations of the eigenmodes n of the nanowire on its support. The strong $n = 4$ mode at 422 kHz was used for the excitation of the ion. The black dashed lines correspond to eigenfrequencies obtained from theoretical modeling. (b) Histograms of arrival times of resonance-fluorescence photons of the ions correlated with the periodic drive signal of the nanowire under near-resonant (top) and far off-resonant (bottom) motional excitation by the mechanical oscillator. The black dashed lines represent fits of the data.

oscillation of the nanowire and its support. From the spectrum, the quality factor of the resonator on this combined resonance is estimated to be $Q = 117$.

To demonstrate the coupling of the ion-nanowire system, the motional excitation of the ions by a resonant drive of the mechanical oscillator was explored. For this purpose, the piezo frequency was set to the mechanical resonance of the nanowire at 422 kHz. The center-of-mass oscillation frequency of the ion along the longitudinal trap axis was matched by suitably adjusting the static potentials applied to the electrodes. Close to resonance between the nanowire and ion oscillations, both systems coupled efficiently. This resulted in a transfer of energy from the mechanical oscillator to the ion [27] which manifested itself as a blurring of the ion in the fluorescence images

due to its motional excitation [compare bottom with top image in Fig. 1(b)].

To quantify the motional excitation of the ions, a photon-correlation method was employed. During driven oscillations in the trap, the ions experienced periodically varying Doppler shifts with respect to the cooling lasers modulating their resonance fluorescence [47–49]. The arrival times of fluorescence photons on a photomultiplier tube (PMT) were correlated with the periodic piezo drive at f_{drive} using lock-in detection. Maximum ion velocities v_{max} were then extracted by fitting the time-resolved fluorescence profiles $R(t)$ to the expression

$$R(t) = R_0 \frac{(\Gamma/2)^2}{(\Gamma/2)^2 + [\delta - k v_{\mathrm{max}} \cos(\omega t - \varphi)]^2}, \quad (1)$$

where R_0 is the scattering rate on resonance, Γ the natural linewidth of the transition, δ the detuning of the cooling-laser frequency from resonance for an ion at rest, k is the projection of the wave vector of the laser on the direction of motion of the ion, ω is the oscillation frequency, and φ is a phase determined by the initial conditions of the experiment.

Figure 2(b) shows typical time-resolved fluorescence traces for a near-resonant ($f_{\mathrm{drive}} = 421.0$ kHz, top panel) and far off-resonant ($f_{\mathrm{drive}} = 366.2$ kHz, bottom panel) mechanical drive of the ion motion. In these measurements, a voltage $V_{\mathrm{nw}} = 1.2$ V was applied to the nanowire which was sinusoidally driven with a voltage $V_{\mathrm{piezo}} = 5$ V applied to the piezo. The equilibrium distance between the ion and the nanowire was determined from a fit to experimental data as $d \approx 300$ μm (see below). A modulation of the fluorescence yield can be seen at strong excitation while the signal appears largely unmodulated with an off-resonant drive. The maximum ion velocities v_{max} during oscillation were determined to be $v_{\mathrm{max}} = 55.3 \pm 2.0$ m/s and 1.1 ± 2.2 m/s for the examples shown at the top and bottom in Fig. 2(b), respectively.

To confirm that the motion of the ion was indeed a consequence of the mechanical action of the nanowire rather than a spurious excitation from oscillating stray electric fields in the experiment, the frequency of the voltage applied to the piezo to drive the nanowire was scanned across the strong mechanical resonance at 422 kHz, observed in Fig. 2(a). In each scan step, the oscillation frequency of the ion along the z axis was matched to the piezo drive frequency with an accuracy of ± 0.2 kHz by adjusting the static trapping potentials. The motional excitation of the trapped ions was quantified as above by measuring their maximum velocities v_{max} using the photon correlation method. Figure 3 shows the squared maximum velocities v_{max}^2 of the ions (red crosses), which are proportional to their maximum kinetic energy in the trap, in relation to the piezo drive frequency and superimposed onto the mechanical drive spectrum of the nanowire (orange trace). The measurement was performed with a string of two ions.

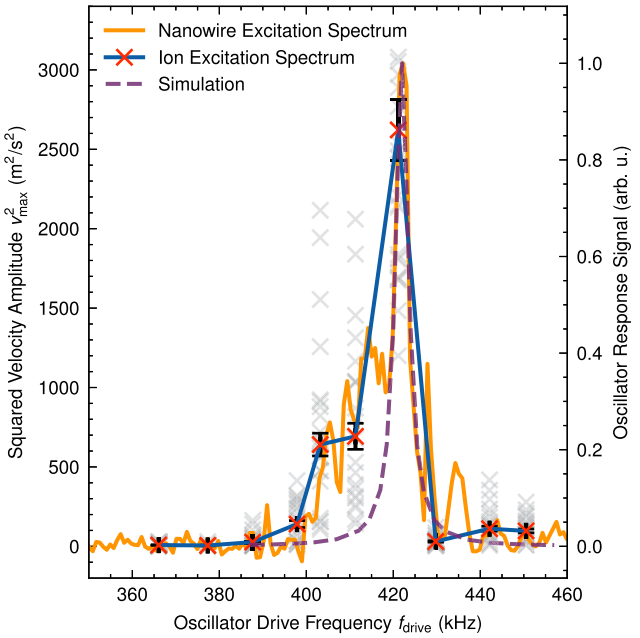

FIG. 3. Squared maximum velocities v_{max}^2 (data points) of two trapped ions over a range of drive frequencies f_{drive} applied to the nanowire. Light gray crosses represent individual measurements, red crosses their average, and the error bars a combination of the fit errors for determining v_{max} from time-resolved ion-fluorescence profiles and the statistical standard error of the mean of all measurements. Orange trace: mechanical excitation spectrum of the nanowire of Fig. 2(a). Motional excitation of the ion could only be observed around the mechanical resonance of the nanowire demonstrating the resonant coupling of the two systems. Purple dashed line: classical dynamics simulations of the mechanical ion excitation at the specific experimental parameters. Traces are normalized to the experimental data for comparison.

Figure 3 demonstrates that the motion of the ions is only excited when the piezo resonantly drives the motion of the mechanical oscillator. This direct correlation of the ion motion with the vibration of the nanowire proves the mechanical nature of the ion excitation and thus the successful coupling between the two systems. The dashed purple curve in Fig. 3 represents the results of a classical simulation of the motional excitation of a single ion by the action of the nanowire following the model of Ref. [27]. Briefly, the simulation treats the excitation of the ion by the nanooscillator by approximating the system as two oscillating point charges which interact via their mutual Coulomb repulsion corresponding to the leading term in a multipole-expansion of their mutual electrostatic interactions [50]. The classical equations of motion of the ion were solved using a Velocity-Verlet algorithm [51]. Further details of the modeling are given in Ref. [27] and Supplemental Material [52].

While this simplified treatment cannot capture the asymmetric line shape of the mechanical resonance of

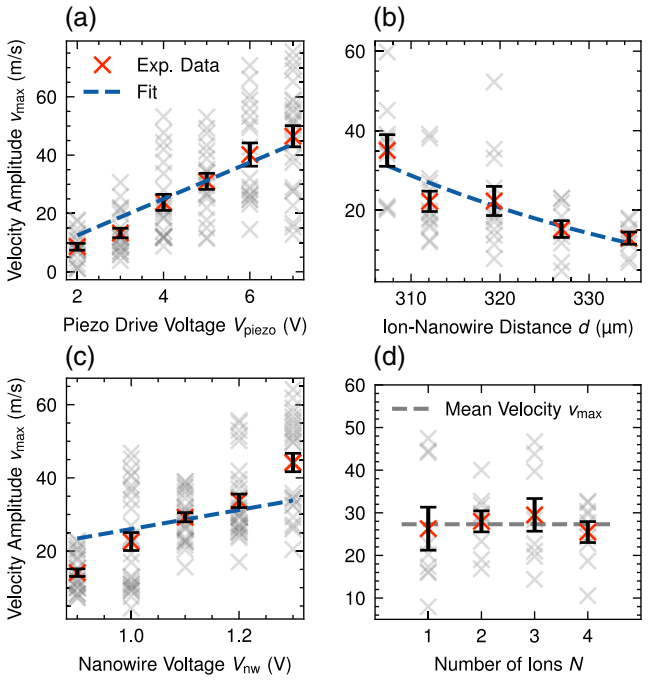

FIG. 4. Maximum ion velocities v_{max} as a function of (a) the voltage V_{piezo} of the piezo actuator driving the nanowire, (b) the ion-nanowire distance d, (c) the voltage V_{nw} applied to the nanowire, and (d) the number of ions in the trap. Dashed lines show the results of a classical simulation of the system. Light gray crosses represent individual measurements, red crosses their average, and the error bars a combination of the fit errors for determining v_{max} from time-resolved ion-fluorescence profiles and the statistical standard error of the mean of all measurements.

the nanowire (which we attribute to overlapping mechanical modes of the entire assembly), it reproduces the general features of the frequency response of the excitation of the ion by the nanooscillator. The modeling also shows that the degree of motional excitation of the ion results from the balance of its continuous cooling by the lasers with the transfer of energy from the nanowire.

To further explore the properties of the coupled system, its response to variations of salient experimental parameters was studied. Figures 4(a) and 4(b) show the maximum velocities of the ions excited by the nanooscillator on the 422 kHz resonance as a function of the piezo drive voltage and the ion-nanowire distance, respectively. As expected, the excitation of the ions is increased with increasing the oscillation amplitude of the nanowire, i.e., with increasing piezo voltage, and decreased with increasing distance between the two subsystems. The dashed blue lines show least-squares fits of the classical simulation model [27] to the combined data (see Supplemental Material [52] for details of the procedure). The good agreement illustrates that the dynamics of the system is well captured by the theory.

Figure 4(c) shows the variation of the ion's maximum velocity subject to the resonant drive at varying bias voltages applied to the nanowire. The coupling, and hence the motional excitation of the ion, seem to increase linearly with the charge on the nanowire in the voltage range studied. The dashed blue line shows the results of simulations assuming the same parameters as used in Figs. 4(a) and 4(b). While the simulation captures the trend correctly, it predicts a weaker dependence of the ion velocity on the nanowire voltage than observed experimentally. We attribute this discrepancy to the simplifications of the model, in particular to the neglect of asymmetries in the charge distribution of the nanowire on its holder, i.e., to higher-order terms in the multipole expansion of the electrostatic interaction between the two systems, and to the possible presence of stray electric fields in the setup.

Finally, Fig. 4(d) shows the dependence of the maximum velocity on the number of trapped ions subjected to the drive by the nanooscillator. It can be observed that the kinetic energy imparted to each ion is independent of the other ions in the trap, consistent with the notion that the center-of-mass motion of the entire ion crystal is excited by the nanowire.

The present study demonstrated the coupling of trapped ions to a nanomechanical oscillator in the classical regime. With the ions cooled close to the Doppler limit and a "heavy" nanowire held at room temperature, we estimate thermal decoherence rates of order $\gamma_{ion} \sim 10$ Hz and $\gamma_{nw} \sim 100$ GHz for the two subsystems, respectively. With an estimated coupling strength g of a few Hz [27], the decoherence rates by far exceed the coupling strength so that the system is firmly operating in the classical regime.

The present results prove the principle feasibility of coupling both systems and lay the foundations for further developing the experiment towards a genuinely quantum hybrid system. As a first step, cooling the ion into the quantum regime of motion in the trap by, e.g., resolved-sideband methods [2] paves the way for generating different quantum states of ion motion through the action of the classical mechanical oscillator as explored theoretically in Ref. [27]. Here, in addition to mechanically driving the ion, the nanowire can be positioned to introduce anharmonicities into the trapping potential which can be used for the deterministic generation of non-Gaussian motional states of the ion.

To realize a full quantum hybrid system, the nanowire also needs to be cooled to the quantum regime ensuring sufficiently small thermal decoherence rates. This is not achievable with the present "heavy" nanowire, but could be reachable using low-mass, high-Q oscillators like carbon nanotubes in a dilution-refrigerator environment. As shown theoretically in Ref. [27], under these conditions the exchange of discrete phonons between the two subsystems, their quantum entanglement, and their mutual sympathetic cooling become a possibility.

A resonant motional excitation of the ions can of course also be achieved by simply applying a resonant "tickle" RF

to any of the trap electrodes [42]. Indeed, this technique was also used here to independently determine the ion motional frequencies. By contrast, the nanowire in the present experiments effectively acted as a miniaturized, movable, oscillating trap electrode which resonantly couples to the ions in a controllable fashion. Moreover, as demonstrated here the nanowire drives the ion through its mechanical motion rather than purely electrically. Thus, the distinct features of using nanomechanical oscillators for the laserless manipulation of the ions are their capability to precisely shape the trapping potentials, to easily control the mechanical drive strength, and to accurately adjust the coupling strength by varying their position and voltage.

In addition, the present experiment could be further developed in a variety of directions. Driving the nanowire optically may offer more localized control over its oscillation without driving other parts of the assembly such as its holder [43]. Integrated optics could improve the optical *in situ* readout of the nanowire resonances. Lastly, the use of other types of nanomechanical oscillators such as membranes as candidates for the mechanical quantum-state manipulation of ions could be investigated.

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[1] F. G. Major, V. N. Gheorghe, and G. Werth, *Charged Particle Traps* (Springer, Berlin & Heidelberg, 2005).
[2] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, Quantum dynamics of single trapped ions, Rev. Mod. Phys. **75**, 281 (2003).
[3] H. Häffner, C. F. Roos, and R. Blatt, Quantum computing with trapped ions, Phys. Rep. **469**, 155 (2008).
[4] C. Flühmann, T. L. Nguyen, M. Marinelli, V. Negnevitsky, K. Mehta, and J. P. Home, Encoding a qubit in a trapped-ion mechanical oscillator, Nature (London) **566**, 513 (2019).
[5] J. M. Pino, J. M. Dreiling, C. Figgatt, J. P. Gaebler, S. A. Moses, M. S. Allman, C. H. Baldwin, M. Foss-Feig, D. Hayes, K. Mayer, C. Ryan-Anderson, and B. Neyenhuis, Demonstration of the trapped-ion quantum CCD computer architecture, Nature (London) **592**, 209 (2021).
[6] B. P. Lanyon, C. Hempel, D. Nigg, M. Müller, R. Gerritsma, F. Zähringer, P. Schindler, J. T. Barreiro, M. Rambach, G. Kirchmair, M. Hennrich, P. Zoller, R. Blatt, and C. F. Roos, Universal digital quantum simulation with trapped ions, Science **334**, 57 (2011).
[7] R. Blatt and C. F. Roos, Quantum simulations with trapped ions, Nat. Phys. **8**, 277 (2012).
[8] J. Whitlow, Z. Jia, Y. Wang, C. Fang, J. Kim, and K. R. Brown, Quantum simulation of conical intersections using trapped ions, Nat. Chem. **15**, 1509 (2023).
[9] A. D. Ludlow, M. M. Boyd, J. Ye, E. Peik, and P. O. Schmidt, Optical atomic clocks, Rev. Mod. Phys. **87**, 637 (2015).
[10] S. Willitsch, Coulomb-crystallised molecular ions in traps: Methods; applications; prospects, Int. Rev. Phys. Chem. **31**, 175 (2012).
[11] Z. Meir, T. Sikorsky, R. Ben-shlomi, N. Akerman, Y. Dallal, and R. Ozeri, Dynamics of a ground-state cooled ion colliding with ultracold atoms, Phys. Rev. Lett. **117**, 243401 (2016).
[12] S. Willitsch, Chemistry with controlled ions, Adv. Chem. Phys. **162**, 307 (2017).
[13] S. Willitsch, Ion-atom hybrid systems, Proc. Int. Sch. Phys. Enrico Fermi **189**, 255 (2015).
[14] M. Tomza, K. Jachymski, R. Gerritsma, A. Negretti, T. Calarco, Z. Idziaszek, and P. S. Julienne, Cold hybrid ion-atom systems, Rev. Mod. Phys. **91**, 035001 (2019).
[15] D. M. Meekhof, C. Monroe, B. E. King, W. M. Itano, and D. J. Wineland, Generation of nonclassical motional states of a trapped atom, Phys. Rev. Lett. **76**, 1796 (1996).
[16] R. Srinivas, C. M. Löschnauer, M. Malinowski, A. C. Hughes, R. Nourshargh, V. Negnevitsky, D. T. C. Allcock, S. A. King, C. Matthiesen, T. P. Harty, and C. J. Ballance, Coherent control of trapped-ion qubits with localized electric fields, Phys. Rev. Lett. **131**, 020601 (2023).
[17] M. Johanning, A. Braun, N. Timoney, V. Elman, W. Neuhauser, and C. Wunderlich, Individual addressing of trapped ions and coupling of motional and spin states using rf radiation, Phys. Rev. Lett. **102**, 073004 (2009).
[18] C. Ospelkaus, U. Warring, Y. Colombe, K. R. Brown, J. M. Amini, D. Leibfried, and D. J. Wineland, Microwave quantum logic gates for trapped ions, Nature (London) **476**, 181 (2011).
[19] K. R. Brown, C. Ospelkaus, Y. Colombe, A. C. Wilson, D. Leibfried, and D. J. Wineland, Coupled quantized mechanical oscillators, Nature (London) **471**, 196 (2011).
[20] M. Harlander, R. Lechner, M. Brownnutt, R. Blatt, and W. Hänsel, Trapped-ion antennae for the transmission of quantum information, Nature (London) **471**, 200 (2011).
[21] D. An, A. M. Alonso, C. Matthiesen, and H. Häffner, Coupling two laser-cooled ions via a room-temperature conductor, Phys. Rev. Lett. **128**, 063201 (2022).
[22] L. Tian and P. Zoller, Coupled ion-nanomechanical systems, Phys. Rev. Lett. **93**, 266403 (2004).
[23] W. K. Hensinger, D. W. Utami, H.-S. Goan, K. Schwab, C. Monroe, and G. J. Milburn, Ion trap transducers for quantum electromechanical oscillators, Phys. Rev. A **72**, 041405(R) (2005).
[24] T. Liu, M. Feng, and K.-L. Wang, Exact solution of quantum dynamics of a cantilever coupling to a single trapped ultracold ion, Commun. Theor. Phys. **47**, 561 (2007).
[25] F. Nicacio, K. Furuya, and F. L. Semião, Motional entanglement with trapped ions and a nanomechanical resonator, Phys. Rev. A **88**, 022330 (2013).
[26] S. Kotler, R. W. Simmonds, D. Leibfried, and D. J. Wineland, Hybrid quantum systems with trapped charged particles, Phys. Rev. A **95**, 022327 (2017).

[27] P. Fountas, M. Poggio, and S. Willitsch, Classical and quantum dynamics of a trapped ion coupled to a charged nanowire, New J. Phys. **21**, 013030 (2019).
[28] N. Daniilidis and H. Häffner, Quantum interfaces between atomic and solid-state systems, Annu. Rev. Condens. Matter Phys. **4**, 83 (2013).
[29] G. Kurizki, P. Bertet, Y. Kubo, K. Molmer, D. Petrosyan, P. Rabl, and J. Schmiedmayer, Quantum technologies with hybrid systems, Proc. Natl. Acad. Sci. U.S.A. **112**, 3866 (2015).
[30] A. Bachtold, J. Moser, and M. I. Dykman, Mesoscopic physics of nanomechanical systems, Rev. Mod. Phys. **94**, 045005 (2022).
[31] A. D. O'Connell, M. Hofheinz, M. Ansmann, R. C. Bialczak, M. Lenander, E. Lucero, M. Neeley, D. Sank, H. Wang, M. Weides, J. Wenner, J. M. Martinis, and A. N. Cleland, Quantum ground state and single-phonon control of a mechanical resonator, Nature (London) **464**, 697 (2010).
[32] J. Chan, T. P. Mayer Alegre, A. H. Sfavi-Naeini, J. T. Hill, A. Krause, S. Gröblacher, M. Aspelmeyer, and O. Painter, Laser cooling of a nanomechanical oscillator into its quantum ground state, Nature (London) **478**, 89 (2011).
[33] J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, Sideband cooling of micromechanical motion to the quantum ground state, Nature (London) **475**, 359 (2011).
[34] M. Rossi, D. Mason, J. Chen, Y. Tsaturyan, and A. Schliesser, Measurement-based quantum control of mechanical motion, Nature (London) **563**, 53 (2018).
[35] F. Tebbenjohanns, M. L. Mattana, M. Rossi, M. Frimmer, and L. Novotny, Quantum control of a nanoparticle optically levitated in cryogenic free space, Nature (London) **595**, 378 (2021).
[36] A. Vochezer, T. Kampschulte, K. Hammerer, and P. Treutlein, Light-mediated collective atomic motion in an optical lattice coupled to a membrane, Phys. Rev. Lett. **120**, 073602 (2018).
[37] A. Jöckel, A. Faber, T. Kampschulte, M. Korppi, M. T. Rakher, and P. Treutlein, Sympathetic cooling of a membrane oscillator in a hybrid mechanical-atomic system, Nat. Nanotechnol. **10**, 55 (2015).
[38] H. Wang and I. Lekavicius, Coupling spins to nanomechanical resonators: Toward quantum spin-mechanics, Appl. Phys. Lett. **117**, 230501 (2020).
[39] G. L. Schmid, C. T. Ngai, M. Ernzer, M. B. Aguilera, T. M. Karg, and P. Treutlein, Coherent feedback cooling of a nanomechanical membrane with atomic spins, Phys. Rev. X **12**, 011020 (2022).
[40] E. Verhagen, S. Deléglise, S. Weis, A. Schliesser, and T. J. Kippenberg, Quantum-coherent coupling of a mechanical oscillator to an optical cavity mode, Nature (London) **482**, 63 (2012).
[41] L. B. Biedermann, R. C. Tung, A. Raman, R. G. Reifenberger, M. M. Yazdanpanah, and C. R. W., Characterization of silver-gallium nanowires for force and mass sensing applications, Nanotechnology **21**, 305701 (2010).
[42] M. Drewsen, A. Mortensen, R. Martinussen, P. Staanum, and J. L. Sørensen, Nondestructive identification of cold and extremely localized single molecular ions, Phys. Rev. Lett. **93**, 243201 (2004).
[43] A. Gloppe, P. Verlot, E. Dupont-Ferrier, A. Siria, P. Poncharal, G. Bachelier, P. Vincent, and O. Arcizet, Bidimensional nano-optomechanics and topological backaction in a non-conservative radiation force field, Nat. Nanotechnol. **9**, 920 (2014).
[44] L. M. de Lépinay, B. Pigeau, B. Besga, P. Vincent, P. Poncharal, and O. Arcizet, A universal and ultrasensitive vectorial nanomechanical sensor for imaging 2d force fields, Nat. Nanotechnol. **12**, 156 (2017).
[45] Comsol multiphysics version 3.5a and 4.3a, www.comsol.com.
[46] B. D. Hauer, C. Doolin, K. S. D. Beach, and J. P. Davis, A general procedure for thermomechanical calibration of nano/micro-mechanical resonators, Ann. Phys. (Amsterdam) **339**, 181 (2013).
[47] D. J. Berkeland, J. D. Miller, J. C. Bergquist, W. M. Itano, and D. J. Wineland, Minimization of ion micromotion in a Paul trap, J. Appl. Phys. **83**, 5025 (1998).
[48] B. Wang, J. W. Zhang, Z. H. Lu, and L. J. Wang, Direct measurement of micromotion speed in a linear quadrupole trap, J. Appl. Phys. **108**, 013108 (2010).
[49] J. Keller, H. L. Partner, T. Burgermeister, and T. E. Mehlstäubler, Precise determination of micromotion for trapped-ion optical clocks, J. Appl. Phys. **118**, 104501 (2015).
[50] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (Wiley, New York, 1999).
[51] Q. Spreiter and M. Walter, Classical molecular dynamics simulation with the velocity verlet algorithm at strong external magnetic fields, J. Comput. Phys. **152**, 102 (1999).
[52] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.133.223201 for a summary of the theoretical modeling as well as of the simulation and fitting procedures, which includes Ref. [53].
[53] L. Hand and J. Finch, *Analytical Mechanics* (Cambridge University Press, Cambridge, England, 2008).