Supplementary Information for Coupling trapped ions to a nanomechanical oscillator

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S1. Ion-nanowire interaction model

The theoretical model and numerical procedures have been outlined in Ref. [27]. The coupling between the ion and the charged nanowire is constituted by the electrostatic interaction between the two subsystems. The ion is represented by a point charge. Similarly, we approximate the charge distribution of the nanowire to that of a point charge close to its tip so that the ion-nanowire interaction assumes the form of a Coulomb potential:

$$\Phi_{\mathrm{IA}} = k_{\mathrm{c}} \frac{q_{\mathrm{ion}} q_{\mathrm{nw}}}{|\vec{r}|}. \tag{S1}$$

This approximation corresponds to the leading term of a multipole-expansion of the electrostatic interaction potential [50] between the ion and the nanowire. Here, $\vec{x}_{\mathrm{ion}}$ and $\vec{x}_{\mathrm{nw}}$ are the positions of the ion and the nanowire tip, respectively. $|\vec{r}| = |\vec{x}_{\mathrm{ion}} - \vec{x}_{\mathrm{nw}}| = \sqrt{(x_{\mathrm{ion}} - x_{\mathrm{nw}})^2 + (y_{\mathrm{ion}} - y_{\mathrm{nw}})^2 + (z_{\mathrm{ion}} - z_{\mathrm{nw}})^2}$ is the distance between the ion and the nanowire tip, $q_{\mathrm{ion}} = 1.602 \times 10^{-19}$ As is the charge of a single ion and $k_{\mathrm{c}} = \frac{1}{4\pi\varepsilon_0}$ is the Coulomb constant. In the case of a mechanically driven motion of the nanowire in the z-direction (the axial direction of the trap), z_{nw} must be replaced with $z_{\mathrm{nw}} \to z_{\mathrm{nw}} + A\cos(\omega_{\mathrm{nw}} t)$ [27] where z_{nw} is now the rest position of the nanowire tip and A and ω_{nw} are its oscillation amplitude and angular frequency, respectively. Equation (S1) thus becomes

$$\Phi_{\mathrm{IA}} = k_{\mathrm{c}} \frac{q_{\mathrm{ion}} q_{\mathrm{nw}}}{\sqrt{(x_{\mathrm{ion}} - x_{\mathrm{nw}})^2 + (y_{\mathrm{ion}} - y_{\mathrm{nw}})^2 + (z_{\mathrm{ion}} - z_{\mathrm{nw}} - A\cos(\omega_{\mathrm{nw}} t))^2}}. \tag{S2}$$

In order to estimate the effective charge q_{nw} of the nanowire in this model, finite-element-method (FEM) simulations have been performed with the Comsol Multiphysics software [48]. Figure S1 shows the simulated electric potential generated by the nanowire at a static voltage $V_{\mathrm{nw}} = 1$ V with its tip positioned in the geometrical center of the trap. This geometry corresponds closely to the conditions in the experiment. As can be seen in figure S1a, the electric potential Φ_{IA} generated by the nanowire is not spherically symmetric and thus not perfectly described by equation (S1) (a corresponding equipotential line is illustrated by the dashed grey circle in Figure S1). Deviations from the ideal point-charge potential can mainly be attributed to the tungsten holder which acts as an electrode of comparable size to the overall dimensions of the trap. However, the potential can still be approximately described by the spherically symmetric point-charge potential in the region above the nanowire close to the symmetry axis of the assembly in the y-direction (dashed orange line in Figure S1a), where also the ions are located in the experiment.

Thus, assuming the point-charge model given by equation (S1), the charge of the nanowire was estimated from a least-square fit to the simulated potential along the dashed orange line in Figure S1a using the fit function

$$\Phi(y) = k_{\mathrm{c}} \frac{q_{\mathrm{ion}} q_{\mathrm{nw}}}{y - d_{\mathrm{offset}}}. \tag{S3}$$

Here, q_{nw} and d_{offset} are fit parameters. d_{offset} is an additional degree of freedom corresponding to an effective offset of the point charge best describing the potential generated by the nanowire. The result is shown in figure S1b which compares the fitted point-charge model (blue solid line) to the numerical potential (dashed orange line). The fitted function agrees well with the numerical potential in the considered region vindicating the application of the point-charge model to describe the dynamics of the present system.

From the fit, the values $q_{\mathrm{nw}} = 1.84 \times 10^{-15}$ As, $d_{\mathrm{offset}} = -33.5$ µm were obtained. The effective charge is proportional to the applied voltage V_{nw} as $q_{\mathrm{nw}} = 1.84 \times 10^{-15} \frac{V_{\mathrm{nw}}}{1\ \mathrm{V}}$ As. This value of d_{offset} was thus included in the simulations and fits reported in this work.

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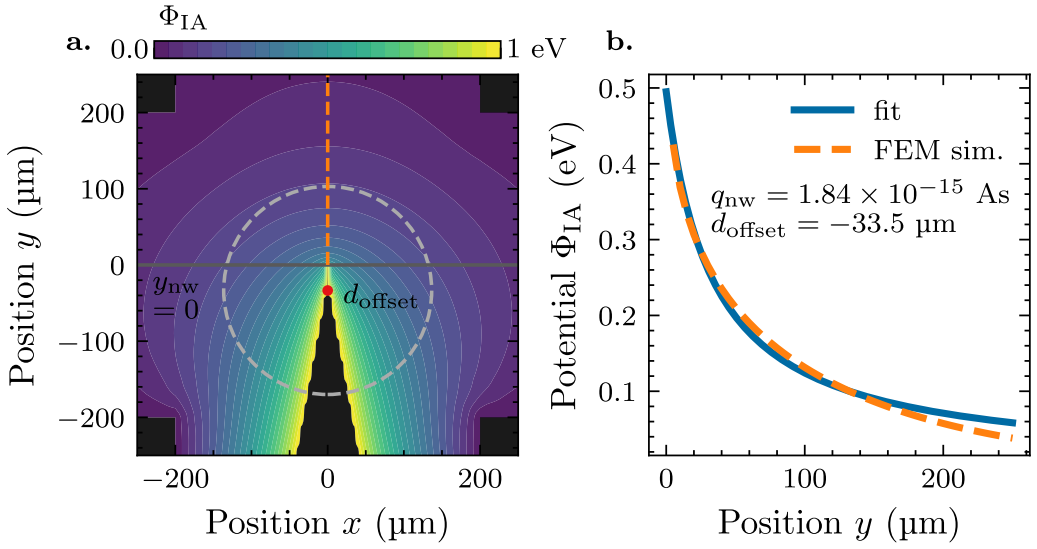

FIG. S1. **FEM simulations of the interaction potential Φ_{IA} generated by the nanowire at $V_{nw} = 1$ V in the center of the trap.** **a**, plot of the ion-nanowire interaction potential in the xy-plane at $z = 0$ with the nanowire and the four rectangular RF and DC electrodes (black). The equipotential lines of the simulated potential show deviations from a perfect point-charge potential with spherical symmetry (indicated by light grey circle). The strongest deviations are found in the region $y < y_{nw} = 0$ (below the dark grey line) due to the presence of the tungsten holder. The potential is closest to that of a point charge along the y-axis (dashed orange line) with the charge located at $d_{offset} = -33.5$ µm (red dot). **b**, Fit of a point-charge model Equation (S3) (blue curve) to the numerical potential along the y-axis (dashed orange curve).

S2. Fit model for experimental data

From Equation (S2), the force F_z along the axial direction of the trap z can be computed as $F_z = -\frac{\partial \Phi_{IA}}{\partial z}$. Taylor expansion to first order around the equilibrium positions of the ion ($\vec{x}_{ion} = (0,0,0)$) and the nanowire ($\vec{x}_{nw} = (x_{nw}, y_{nw}, z_{nw})$) yields

$$F_z \approx -\frac{\varepsilon z_{nw}}{d^3} + \left(\frac{\varepsilon}{d^3} - \frac{3\varepsilon z_{nw}^2}{d^5}\right) z - \left(\frac{\varepsilon}{d^3} - \frac{3\varepsilon z_{nw}^2}{d^5}\right) A\cos(\omega_{nw} t). \quad \text{(S4)}$$

Here, $d = \sqrt{x_{nw}^2 + y_{nw}^2 + z_{nw}^2}$ represents the distance between the equilibrium positions of the ion and the nanowire tip and $\varepsilon = k_c q_{ion} q_{nw}$. The first term in Equation (S4) corresponds to a constant force that shifts the equilibrium position of the ion. The second term modifies the harmonic trapping potential and introduces a shift in the harmonic oscillation frequency of the ion along the z-direction (see below). The third term corresponds to the periodic driving force that couples the nanowire to the ion motion. Including the harmonic pseudopotential $\Phi_{trap} = \frac{1}{2} m\omega_{0,z}^2 z^2$ of the trap [1] with the unperturbed harmonic trap frequency $\omega_{0,z}$, we arrive at the following expression for the force acting on a trapped ion under the action of the oscillating nanowire,

$$F_z = -m\omega_z^2 z - \left(\frac{\varepsilon}{d^3} - \frac{3\varepsilon z_{nw}^2}{d^5}\right) A\cos(\omega_{nw} t). \quad \text{(S5)}$$

Here, $\omega_z = \sqrt{\omega_{0,z}^2 - \frac{\varepsilon}{md^3} + \frac{3\varepsilon z_{nw}^2}{md^5}}$ is the harmonic trap frequency modified by the presence of the nanowire (see also Ref. [27] for a detailed discussion of the effect of the nanowire on the trapping potential).
The resulting equation of motion assumes the form of a damped harmonic oscillator

$$\ddot{z} + \gamma\dot{z} + \omega_z^2 z = -\left(\frac{\varepsilon}{md^3} - \frac{3\varepsilon z_{nw}^2}{md^5}\right) A\cos(\omega_{nw} t). \quad \text{(S6)}$$

where a damping term with constant γ has been added to account for the friction force generated by laser cooling the ion [1]. From the steady-state solution of Equation (S6) [53], the amplitude v_{max} of the ion velocity along the z-direction can be computed as

$$v_{max} = \left|\left(\frac{\varepsilon}{md^3} - \frac{3\varepsilon z_{nw}^2}{md^5}\right)\frac{A}{\gamma}\right|. \quad \text{(S7)}$$

This is the quantity that is determined in the experiments. Consequently, Equation (S7) was used to fit the experimental data in Figures 3 and 4 of the main text with the ion-nanowire distance d and the factor A/γ as fit parameters using a least-squares algorithm.