Nanoscale Magnetic Field Imaging for 2D Materials

Winter School, Arosa, 2022

Sensitivity

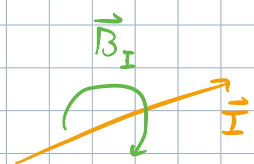
To evaluate sensitivity : Signal-to-noise

Let's take two idealized signals :

- a line of current $\vec{I}$
- a magnetic moment $\vec{m}$

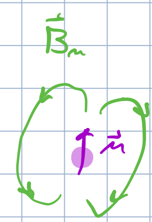
- Current :

$$\vec{B}_I = \frac{\mu_0 \, \vec{I} \times \vec{r}}{2\pi r^2}$$



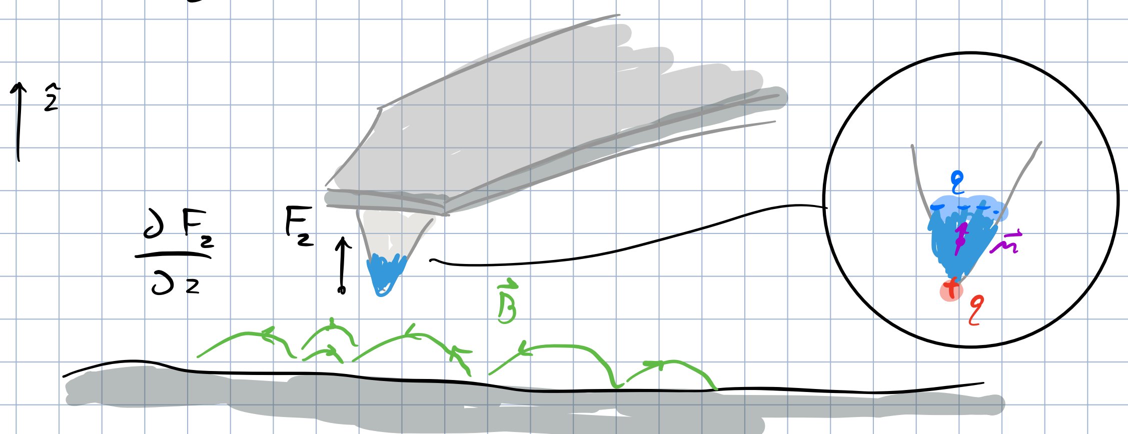
- moment :

$$\vec{B}_m = \frac{\mu_0}{4\pi r^3}\left(\frac{3(\vec{m}\cdot\vec{r})\,\vec{r}}{r^2} - \vec{m} \right)$$



MFM

- Signal



$$F_z = \underbrace{q\,\vec{B}\cdot\hat{z}}_{\text{monopole}} + \underbrace{\vec{\nabla}(\vec{\mu}\cdot\vec{B})\cdot\hat{z}}_{\text{dipole}}$$

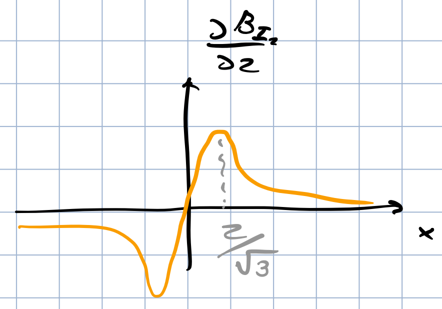
If we assume a mono-pole tip and a measurement of force gradients (Δf):

$$\frac{\partial F_z}{\partial z} = q\,\frac{\partial B_z}{\partial z} \quad \therefore \quad \boxed{\frac{\partial B_z}{\partial z} = \frac{1}{q}\frac{\partial F_z}{\partial z}}$$

The maximum signal at a given height z that we can measure for a line of current:

$$\frac{\partial}{\partial x}\left(\frac{\partial B_{I_z}}{\partial z}\right) = 0$$

$$x = -\frac{z}{\sqrt{3}}\,,\ \frac{z}{\sqrt{3}}$$

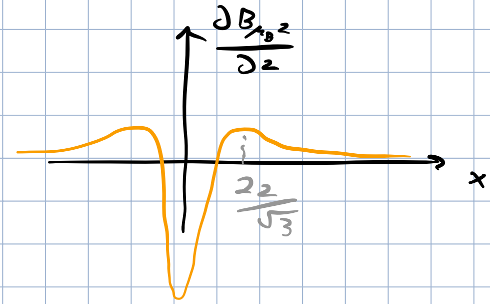


$$\boxed{\left(\frac{\partial B_{I_z}}{\partial z}\right)_{max} = \frac{3\sqrt{3}\,\mu_0\, I}{16\pi\, z^2}} \qquad [T/m]$$

Similarly for a μ_B of magnetic moment:

$$\frac{\partial}{\partial x}\left(\frac{\partial B_{\mu_B z}}{\partial z}\right) = 0$$

$$x = -\frac{2z}{\sqrt{3}}\,,\ 0\,,\ \frac{2z}{\sqrt{3}}$$



$$\boxed{\left(\frac{\partial B_{\mu_B z}}{\partial z}\right)_{max} = \frac{3\mu_0\,\mu_B}{2\pi\, z^4}} \qquad [T/m]$$

- **Noise**

The ultimate noise limit is from the thermal motion of the cantilever:

$$S_F = 4 k_B T \Gamma \quad \leftarrow \text{Fluctuation - Dissipation Theorem}$$

This implies a thermal force noise amplitude that sets a minimum measurable force:

$$F_{min} = \sqrt{4 k_B T \Gamma}$$

For measurements of force gradients done by oscillating the cantilever by z_{rms} and monitoring its resonant frequency, we have:

$$\left(\frac{\partial F}{\partial z}\right)_{min} = \frac{1}{z_{rms}} \sqrt{4 k_B T \Gamma}$$

$30 \; \frac{T}{m\sqrt{Hz}}$
@ 4K

$$\therefore \boxed{\left(\frac{\partial B_z}{\partial z}\right)_{min} = \frac{1}{q\, z_{rms}} \sqrt{4 k_B T \Gamma}} \qquad \left[\frac{T}{m\sqrt{Hz}}\right]$$

We can then see the sensitivity to I or μ_B by writing:

Current Sens.

$$\frac{\left(\frac{\partial B_z}{\partial z}\right)_{min}}{\left(\frac{\partial B_{I_z}}{\partial z}\right)_{max}} \cdot I \propto z^2 \qquad \left[A/\sqrt{Hz} \right]$$

$\mu A/\sqrt{Hz}$ @ 50 nm

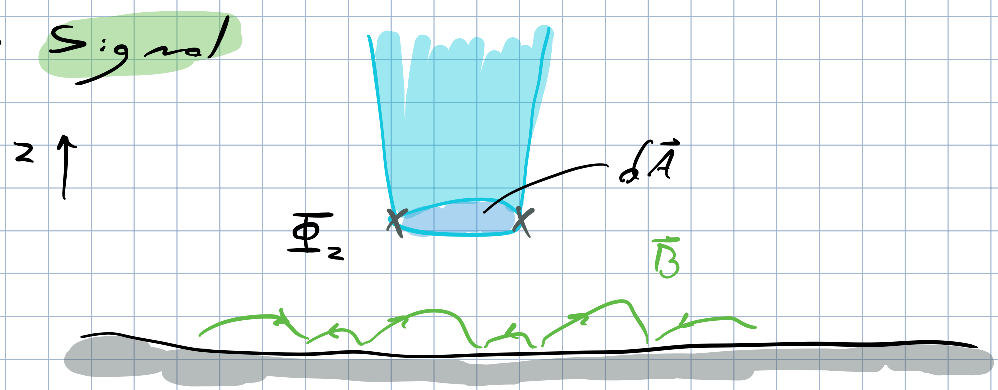
Moment Sens.

$$\frac{\left(\frac{\partial B_z}{\partial z}\right)_{min}}{\left(\frac{\partial B_{\mu_B z}}{\partial z}\right)_{max}} \cdot \mu_B \propto z^4 \qquad \left[\mu_B/\sqrt{Hz} \right]$$

$10^3\ \mu_B/\sqrt{Hz}$ @ 50 nm

SSM

- Signal



$$\boxed{\Phi_z = \int \vec{B} \cdot d\vec{A}}$$

If we now calculate the flux directly above a μ_B of moment:

loop radius

$$\boxed{\left(\Phi_{\mu_B z}\right)_{max} = \frac{\mu_0 \mu_B R^2}{2\left(z^2 + R^2\right)^{3/2}}}$$

$[T \cdot m^2 = Wb]$

- Noise

There are several sources of noise:

- Johnson noise
- shot noise
- 1/f noise ← at low freq
- quantum noise $\longrightarrow$ $\Phi_Q = \sqrt{\hbar L}$

state of the art are ~ 4x this limit

Loop inductance

$$\therefore \boxed{\left(\Phi_z\right)_{min} = \Phi_{noise}} \longrightarrow 50 \sim \Phi_0/\sqrt{Hz}$$

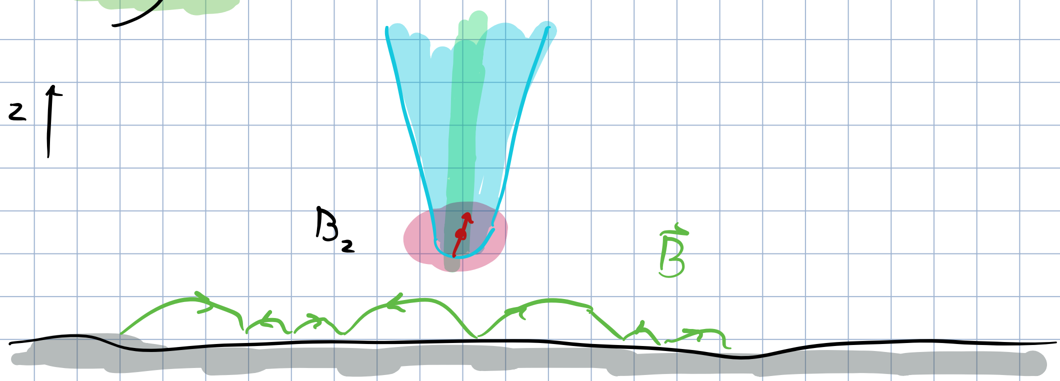
Sensitivity:

Current Sens. $\dfrac{\left(\Phi_z\right)_{min}}{\left(\Phi_{Iz}\right)_{max}} \cdot I$ $\left[A/\sqrt{Hz}\right]$ → $10 \; nA/\sqrt{Hz}$ @ 50nm

Moment Sens. $\dfrac{\left(\Phi_z\right)_{min}}{\left(\Phi_{\mu z_B}\right)_{am}} \cdot \mu_B \propto z^3$ $\left[\mu_B/\sqrt{Hz}\right]$ → $\mu_B/\sqrt{Hz}$ @ 50nm

SNVM

- Signal



By measuring the NV splitting, we measure the magnetic field along the NV axis:

$$\boxed{B_z = \vec{B} \cdot \hat{z}}$$

$$\boxed{\left(B_{Iz}\right)_{max} = \frac{\mu_0 I}{4\pi z}} \qquad [T]$$

$$\boxed{\left(B_{\mu_B z}\right)_{max} = \frac{\mu_0 \mu_B}{2\pi z^3}} \qquad [T]$$

- Noise

SNVM is typically limited by photon shot noise from the optical read-out. Minimum measurable field can be written as:

optical contrast

$$\boxed{(B_z)_{min} = \frac{1}{\gamma \, \varepsilon \sqrt{I_0 \, t_{acq} \, T_2}}}$$

→ $100 \, nT/\sqrt{Hz}$

decoherence

gyromagnetic ratio

count rate

integration

Sensitivity:

Current Sens. $\dfrac{(B_z)_{min}}{(B_{I_z})_{max}} \cdot I \propto z$ $\left[A/\sqrt{Hz} \right]$ → $10 \, nA/\sqrt{Hz}$ @ 25 nm

Moment Sens. $\dfrac{(B_z)_{min}}{(B_{\mu_0 z})_{max}} \cdot \mu_B \propto z^3$ $\left[\mu_B/\sqrt{Hz} \right]$ → $\mu_B/\sqrt{Hz}$ @ 25 nm

Reconstruction of $\vec{J}$ & $\vec{m}$ from $\vec{B}$

Biot - Savart :

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} d^3 \vec{r}\,'$$

For current density $\vec{J} = J_x \hat{x} + J_y \hat{y}$

or magnetization $\vec{m} = m_z \hat{z}$ in 2D

we can write this in k-space :

$$\tilde{B}_z(k_x, k_y, z) = \underbrace{i \frac{1}{2} \mu_0 d\, e^{-kz}}_{g(k,z)} \left[\frac{k_y}{k} \tilde{J}_x(k_x, k_y) - \frac{k_x}{k} \tilde{J}_y(k_x, k_y) \right]$$

w/ $k = \sqrt{k_x^2 + k_y^2}$

and $d \ll z$

film thickness

Continuity equation : $\vec{\nabla} \cdot \vec{J} = 0$

$$\longrightarrow \quad k_x \tilde{J}_x + k_y \tilde{J}_y = 0$$

$$\tilde{J}_y = - \frac{k_x}{k_y} \tilde{J}_x$$

Together :

$$\tilde{B}_z = i g \frac{\tilde{J}_x}{k} \left(k_y + \frac{k_x^2}{k_y} \right) = i g \frac{\tilde{J}_x}{k_y} k$$

$$\boxed{\tilde{J}_x = -\frac{i k_y \tilde{B}_z}{k g} \qquad \tilde{J}_y = \frac{i k_x \tilde{B}_z}{k g}}$$

Magnetization :

$$\vec{J} = \vec{\nabla} \times \vec{m}$$

$$\therefore \quad J_x = \frac{\partial m_z}{\partial y}, \quad J_y = -\frac{\partial m_z}{\partial x}$$

$$\tilde{J}_x = -i k_y \tilde{m}_z, \quad \tilde{J}_y = i k_x \tilde{m}_z$$

$$\therefore \quad \boxed{\tilde{m}_z = \frac{\tilde{B}_z}{k g}}$$

$$\boxed{\text{Current density}} \qquad\qquad \boxed{\text{Magnetization}}$$

$$\tilde{B}_z = -i \frac{1}{2} \mu_0 d e^{-kz} \frac{k}{k_x} \tilde{J}_y \qquad\qquad \tilde{B}_z = \frac{1}{2} \mu_0 d e^{-kz} k \tilde{m}_z$$

Derivative along z :

$$\frac{\partial \tilde{B}_z}{\partial z} \propto e^{-kz} \frac{k^2}{k_x} \qquad\qquad \frac{\partial \tilde{B}_z}{\partial z} \propto e^{-kz} k^2$$

$$k \propto k_x \propto \frac{1}{\lambda}$$ $\longrightarrow$ Feature size

Normalized to distance : $\frac{\lambda}{2}$

$$\tilde{B}_z \propto e^{-\frac{z}{\lambda}} \qquad \tilde{B}_z \propto \frac{1}{z}\left(\frac{z}{\lambda}\right) e^{-\frac{z}{\lambda}}$$

$$\frac{\partial \tilde{B}_z}{\partial z} \propto \frac{1}{z}\left(\frac{z}{\lambda}\right) e^{-\frac{z}{\lambda}} \qquad \frac{\partial \tilde{B}_z}{\partial z} \propto \frac{1}{z^2}\left(\frac{z}{\lambda}\right)^2 e^{-\frac{z}{\lambda}}$$