

# Nanomagnetic Imaging using Scanning Probe Microscopy Techniques

PhD Course, EPFL, 2020

## Sensitivity

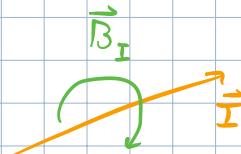
To evaluate sensitivity : **Signal - to - noise**

Let's take two localized signals :

- a line of current  $\vec{I}$
- a magnetic moment  $\vec{m}$

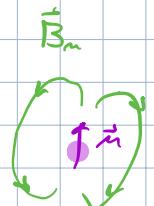
- Current :

$$\vec{B}_I = \frac{\mu_0 \vec{I} \times \vec{r}}{2\pi r^2}$$



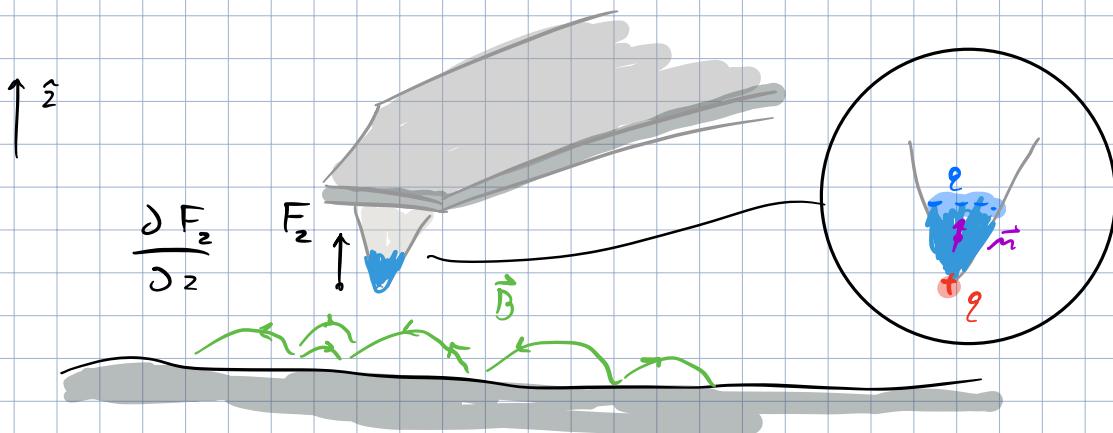
- moment :

$$\vec{B}_m = \frac{\mu_0}{4\pi r^3} \left( \frac{3(\vec{m} \cdot \vec{r})\vec{r}}{r^2} - \vec{m} \right)$$



**MEM**

- Signal



$$F_z = \mu_0 \vec{B} \cdot \hat{z} + \vec{\nabla}(\vec{m} \cdot \vec{B}) \cdot \hat{z}$$

↳ monopole
↳ dipole

If we assume a monopole tip and a measurement of force gradients ( $\Delta F$ ):

$$\frac{\partial F_z}{\partial z} = \mu_0 \frac{\partial B_z}{\partial z}$$

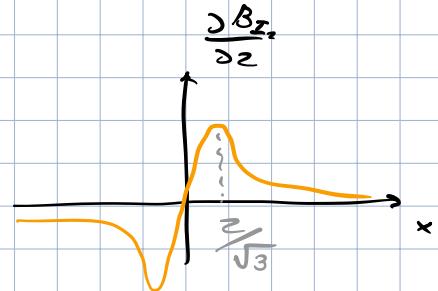
$\therefore$

$$\frac{\partial B_z}{\partial z} = \frac{1}{\mu_0} \frac{\partial F_z}{\partial z}$$

The maximum signal at a given height  $z$  what we can measure for a line of current:

$$\frac{\partial}{\partial x} \left( \frac{\partial B_{Iz}}{\partial z} \right) = 0$$

$$x = -\frac{z}{\sqrt{3}}, \frac{z}{\sqrt{3}}$$



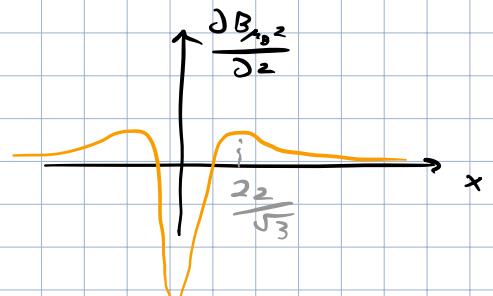
$$\left( \frac{\partial B_{Iz}}{\partial z} \right)_{\max} = \frac{3\sqrt{3}\mu_0 I}{16\pi z^2}$$

$$\left[ \frac{T}{m} \right]$$

Similarly for a  $\mu_B$  of magnetic moment:

$$\frac{\partial}{\partial x} \left( \frac{\partial B_{\mu_B z}}{\partial z} \right) = 0$$

$$x = -\frac{2z}{\sqrt{3}}, 0, \frac{2z}{\sqrt{3}}$$



$$\left( \frac{\partial B_{\mu_B z}}{\partial z} \right)_{\max} = \frac{3\mu_0 \mu_B}{2\pi z^4}$$

$$\left[ \frac{T}{m} \right]$$

- Noise

The ultimate noise limit is from the thermal motion of the container:

$$S_F = 4k_B T \Gamma \leftarrow \text{Fluctuation-Dissipation Theorem}$$

This implies a thermal force noise amplitude that sets a minimum measurable force:

$$F_{\min} = \sqrt{4k_B T \Gamma}$$

For measurements of force gradients done by oscillating the container by  $\zeta_{\text{rms}}$  and monitoring its resonant frequency, we have:

$$\left( \frac{\partial F}{\partial z} \right)_{\min} = \frac{1}{\zeta_{\text{rms}}} \sqrt{4k_B T \Gamma} \rightarrow 30 \frac{\text{T}}{\text{nJHz}} @ 4K$$

$$\therefore \left( \frac{\partial B_z}{\partial z} \right)_{\min} = \frac{1}{2\zeta_{\text{rms}}} \sqrt{4k_B T \Gamma} \left[ \frac{\text{T}}{\text{nJHz}} \right]$$

We can then see the sensitivity to  $I$  or  $\mu_B$  by writing:

Current Sens.

$$\frac{\left(\frac{\partial B_z}{\partial z}\right)_{\min}}{\left(\frac{\partial B_{I_2}}{\partial z}\right)_{\max}} \cdot I \propto z^2$$

$\left[ \frac{A}{\sqrt{Hz}} \right] \xrightarrow{\text{at } 50\text{ nm}} \frac{m}{\sqrt{Hz}}$

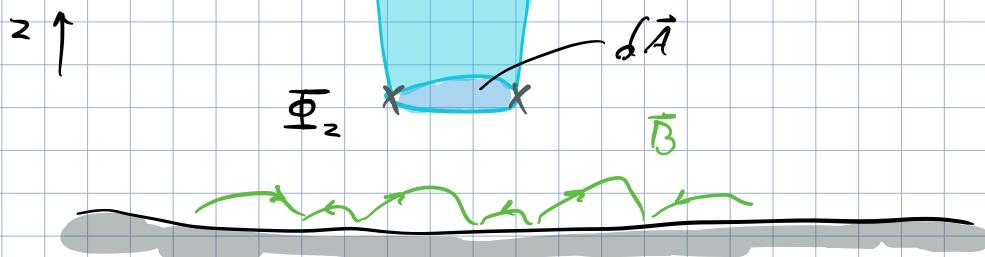
Moment Sens.

$$\frac{\left(\frac{\partial B_z}{\partial z}\right)_{\min}}{\left(\frac{\partial B_{\mu_B z}}{\partial z}\right)_{\max}} \cdot \mu_B \propto z^4$$

$\left[ \frac{\mu_B}{\sqrt{Hz}} \right] \xrightarrow{\text{at } 50\text{ nm}} 10^3 \frac{\mu_B}{\sqrt{Hz}}$

### SSM

- Signal



$$\Phi_z = \int \vec{B} \cdot d\vec{A}$$

If we now calculate the flux directly above a  $\mu_B$  of moment:

$$\left( \frac{\Phi_{Bz}}{B_z} \right)_{\text{max}} = \frac{\mu_0 \mu_B R^2}{2(z^2 + R^2)^{3/2}}$$

loop radiuses

$$[ F \cdot m^2 = W_b ]$$

### Noise

There are several sources of noise:

- Johnson noise

- shot noise

- 1/f noise

- quantum noise  $\rightarrow \Phi_Q = \sqrt{k_L}$

at low freq

state of  
the art  
are  $\sim 4x$   
this limit

Logo indicates

$$\therefore \left( \frac{\Phi_z}{B_z} \right)_{\text{min}} = \Phi_{\text{noise}}$$

$$\rightarrow S_0 \sim \frac{\Phi_0}{\sqrt{Hz}}$$

Sensitivity:

Current  
Sens.

$$\frac{\left( \frac{\Phi_z}{B_z} \right)_{\text{min}}}{\left( \frac{\Phi_z}{B_z} \right)_{\text{max}}} \cdot I$$

$$\left[ \frac{A}{\sqrt{Hz}} \right]$$

$10 \frac{nA}{\sqrt{Hz}} @ 50nm$

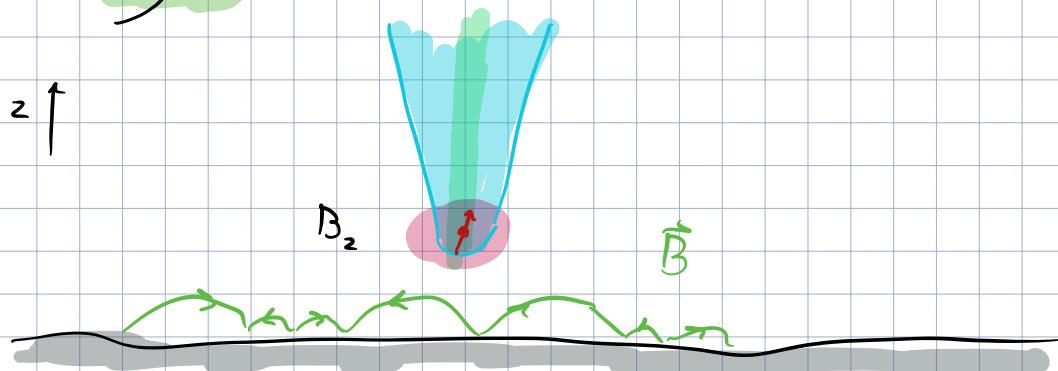
Moment  
Sens.

$$\frac{\left( \frac{\Phi_z}{B_z} \right)_{\text{min}}}{\left( \frac{\Phi_{Bz}}{B_z} \right)_{\text{max}}} \cdot \mu_B \propto z^3 \left[ \frac{\mu_B}{\sqrt{Hz}} \right]$$

$\frac{\mu_B}{\sqrt{Hz}} @ 50nm$

# SNUM

- Signal



By measuring the NV splitting, we measure the magnetic field along the NV axis:

$$B_z = \vec{B} \cdot \hat{z}$$

$$(B_{z_2})_{\max} = \frac{\mu_0 I}{4\pi z} \quad [\text{T}]$$

$$(B_{B_2})_{\max} = \frac{\mu_0 M_0}{2\pi z^3} \quad [\text{T}]$$

## Noise

SNRM is typically limited by photon shot noise from the optical readout.

Minimum measure 61° Fold con 60°

written as :

- factor as :

$$(B_z)_{\text{osc}} = \frac{I}{\gamma \sum \sqrt{I_0 + \omega_q T_2}}$$

optical contrast

gyromagnetic ratio

coant rate

integretion

decoherence

$\rightarrow 100 \text{ nT} / \sqrt{\text{Hz}}$

## Sensitivity:

$$\text{Current Sors.} \quad \frac{(\beta_z)_{\min}}{(\beta_{Iz})_{\max}} \cdot I \propto z \quad \left[ \frac{A}{\sqrt{\text{Hz}}} \right] \xrightarrow{\sim} 10 \text{ mA} / \sqrt{\text{Hz}}$$

$$\frac{(\beta_2)_{\min}}{(\beta_{M_0,2})_{\max}} \cdot \mu_B \propto 2^3 \left[ \frac{\mu_B}{\sqrt{H_2}} \right] \rightarrow \frac{\mu_B}{\sqrt{H_2}}$$

# Reconstruction of $\vec{J}$ & $\vec{m}$ from $\vec{B}$

Biot - Savart :

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} d^3 r'$$

For current density  $\vec{J} = J_x \hat{x} + J_y \hat{y}$

or magnetization  $\vec{m} = M_z \hat{z}$  in 2D

we can write this in k-space :

$$\tilde{B}_z(k_x, k_y, z) = i \underbrace{\frac{1}{2} \mu_0 d e^{-kz}}_{g(k, z)} \left[ \frac{k_y}{k} \tilde{J}_x(k_x, k_y) - \frac{k_x}{k} \tilde{J}_y(k_x, k_y) \right]$$

$$\text{w/ } k = \sqrt{k_x^2 + k_y^2}$$

and  $d \ll z$

(film thickness)

Continuity equation :  $\vec{\nabla} \cdot \vec{J} = 0$

$$\rightarrow k_x \tilde{J}_x + k_y \tilde{J}_y = 0$$

$$\tilde{J}_y = - \frac{k_x}{k_y} \tilde{J}_x$$

Together :

$$\tilde{B}_z = i g \frac{\tilde{J}_x}{k} \left( k_y + \frac{k_x^2}{k_y} \right) = i g \frac{\tilde{J}_x}{k_y} k$$

$$\tilde{J}_x = - \frac{i k_y \tilde{B}_z}{k g}$$

$$\tilde{J}_y = \frac{i k_x \tilde{B}_z}{k g}$$

Magneto: zation :

$$\vec{J} = \vec{\nabla} \times \vec{m}$$

↪ ∴  $J_x = \frac{\partial M_z}{\partial y}$ ,  $J_y = - \frac{\partial M_z}{\partial x}$

$$\tilde{J}_x = -i k_y \tilde{M}_z, \quad \tilde{J}_y = i k_x \tilde{M}_z$$

∴  $\tilde{M}_z = \frac{\tilde{B}_z}{k g}$

Current density

Magneto: zation

$$\tilde{B}_z = -i \frac{1}{2} \mu_0 d e^{-k_z k} \frac{k}{k_x} \tilde{J}_y$$

$$\tilde{B}_z = \frac{1}{2} \mu_0 d e^{-k_z k} k \tilde{M}_z$$

Derivatives along z :

$$\frac{\partial \tilde{B}_z}{\partial z} \propto e^{-k_z} \frac{k^2}{k_x}$$

$$\frac{\partial \tilde{B}_z}{\partial z} \propto e^{-k_z} k^2$$

$$k \propto k_x \propto \frac{1}{\lambda} \quad \xrightarrow{\text{Feature Size}}$$

Normalized to distance :  $\frac{\lambda}{z}$

$$\tilde{\beta}_2 \propto e^{-\frac{z^2}{\lambda}}$$

$$\tilde{\beta}_2 \propto \frac{1}{z} \left(\frac{z}{\lambda}\right) e^{-\frac{z^2}{\lambda}}$$

$$\frac{\partial \tilde{\beta}_2}{\partial z} \propto \frac{1}{z} \left(\frac{z}{\lambda}\right) e^{-\frac{z^2}{\lambda}}$$

$$\frac{\partial \tilde{\beta}_2}{\partial z} \propto \frac{1}{z^2} \left(\frac{z}{\lambda}\right)^2 e^{-\frac{z^2}{\lambda}}$$