Time-resolved nonlinear coupling between orthogonal flexural modes of a pristine GaAs nanowire - Supplementary Information

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I. EQUATIONS OF MOTION

To derive the equations of motion for the two modes we treat the nanowire as an isotropic inextensible singly clamped Euler-Bernoulli beam. Following the approach used by Crespo da Silva and Glynn [1,2] we obtain two nonlinear equations describing the flexural vibrations of the beam, for the displacement $\tilde{u}$:

$$\begin{aligned} m\ddot{\tilde{u}} + \tilde{\eta}_1\dot{\tilde{u}} + D_1\tilde{u}'''' + D_1\left(\tilde{u}'\left(\tilde{u}'\tilde{u}'' + \tilde{w}'\tilde{w}''\right)'\right)' \\ + \Bigg((D_1 - D_2)\left[\tilde{w}''\int_L^s \tilde{u}''\tilde{w}''\,ds - \tilde{w}'''\int_0^s \tilde{u}''\tilde{w}'\,ds\right] \\ - \frac{(D_1-D_2)^2}{D_k}\left[\tilde{w}''\int_0^s\int_L^s \tilde{u}''\tilde{w}''\,ds\,ds\right]'\Bigg)' \\ +\frac{1}{2}m\left[\tilde{u}'\int_L^s \frac{\partial^2}{\partial\tilde{t}^2}\int_0^s\left(\tilde{u}'^2 + \tilde{w}'^2\right)\,ds\,ds\right]' = \tilde{Q}_1 - \left(\tilde{u}'\int_L^s \tilde{Q}_2\,ds\right)' \end{aligned} \quad (1)$$

and a symmetric one for the displacement $\tilde{w}$. Here the dots and primes stand for derivative in time $\tilde{t}$ and in arc length s respectively, m is the mass per unit length ($= \rho d_1 d_2$ with d_1 and d_2 dimensions of the cross section and ρ the density), $\tilde{\eta}_{1,2}$ the damping coefficient, $D_{1,2}$ the bending, and D_k the torsional stiffnesses of the beam, and $\tilde{Q}_{1,2}$ the generalized forces along the two directions. We define $\tilde{F}_1 = \tilde{Q}_1 - \left(\tilde{u}'\int_L^s \tilde{Q}_2\,ds\right)'$.

Eq. 1 can be rewritten in a dimensionless form [3] substituting $u = \tilde{u}/d_1, w = \tilde{w}/d_1, x = s/L, \eta_1 = \tilde{\eta}_1 L^4/(D_1\tau)$ and scaling time with $\tau = L^2\sqrt{m/D_1}$.

Applying the Galerkin method for the first mode in the two directions $u(x,t) = a(t)\xi(x)$ and $w(x,t) = b(t)\xi(x)$ with $\xi(x)$ the first flexural mode shape, equal in both directions, we obtain:

$$\begin{aligned} \ddot{a} + \omega_1^2 a + \eta_1\dot{a} + \alpha\Big(\frac{d_1}{L}\Big)^2 a^3 + \Big(\alpha + \Big(1 - \frac{D_2}{D_1}\Big)\beta_1 - \frac{D_1}{D_k}\Big(1 - \frac{D_2}{D_1}\Big)^2\beta_2\Big)\Big(\frac{d_1}{L}\Big)^2 ab^2 \\ + \gamma\Big(\frac{d_1}{L}\Big)^2 a(\dot{a}^2 + a\ddot{a} + \dot{b}^2 + b\ddot{b}) = \varepsilon F_1 \end{aligned} \quad (2)$$

where F_1 is the scaled dimensionless version of $\tilde{F}_1$, and

$$\alpha = \int_0^1 \xi(x)\Big(\xi'(x)\big(\xi'(x)\xi''(x)\big)'\Big)' \, dx = 40.41 \tag{3}$$

$$\begin{aligned}\beta_1 = &\left[\int_0^1 \xi(x)\Big(\xi''(x)\int_1^{x_1}\Big(\xi''(x)\Big)^2 dx_1\Big)' \, dx\right] \\ &- \left[\int_0^1 \xi(x)\Big(\xi'''(x)\int_0^{x_1}\xi''(x)\xi'(x)\, dx_1\Big)' \, dx\right] = -20.11\end{aligned} \tag{4}$$

$$\beta_2 = \int_0^1 \xi(x)\Big(\Big(\xi''(x)\int_0^{x}\int_1^{x_1}\Big(\xi''(x)\Big)^2 dx_2\, dx_1\Big)'\Big)' \, dx = 16.60 \tag{5}$$

$$\gamma = \int_0^1 \xi(x)\Big(\xi'(x)\int_0^{x}\int_1^{x_1}\big(\xi'(x_2)\big)^2 dx_2\, dx_1\Big)' \, dx = 4.60 \tag{6}$$

$$\varepsilon = \int_0^1 \xi(x)\, dx = 0.78 \tag{7}$$

From our COMSOL simulations, the ~ 0.5 % difference in the cross section of the nanowire is already enough to produce a frequency splitting of the two perpendicular modes similar to what is observed, such that $D_1/D_2 \simeq 1.01$. We also consider the beam to have high torsional rigidity compared to the flexural rigidity, such that $D_k \gg D_{1,2}$. Finally, we find that nonlinear damping is negligible by evaluating the critical frequency at which the bistability of the Duffing regime starts to occur. As a result, we obtain the final simplified equation of motion in one direction:

$$\ddot{a} + \omega_1^2 a + \eta_1 \dot{a} + \alpha\Big(\frac{d_1}{L}\Big)^2 a^3 + \alpha\Big(\frac{d_1}{L}\Big)^2 ab^2 = \varepsilon F_1 \tag{8}$$

Note that, from equation 1 the coupling coefficient with dimensions would be defined as:

$$\tilde{\alpha}_1 \equiv \frac{D_1}{mL}\int_0^L \xi(s)\Big(\xi'(s)\big(\xi'(s)\xi''(s)\big)'\Big)' \, ds \tag{9}$$

II. MECHANICAL LOGIC

Due to the Duffing nonlinearity, when sweeping the driving amplitude at fixed frequency f_2, we observe a high jump between two levels in the response amplitude of mode 2, at a critical driving amplitude. These two levels in the response are used to encode logical 0 and 1 output states[4,5]. The two inputs correspond to two signal voltages which are summed and subsequently applied to the driving PZT. Logical 0 and 1 input states are defined by low and high driving voltages, respectively (See Fig. 1a). As shown in Fig. 1b (upper panel), we obtain a high response when one or both inputs are high (01, 10, or 11) and a low response when both input signals are low (00). This is therefore a realization of a logical OR gate. This OR gate is converted into a NOR gate by taking as output the response amplitude of mode 1 at f_1 (lower panel Fig. 1b). When mode 2 is at the low level (for input 00) there is almost no interaction between the two modes and we have the maximum response (logical 1) of mode 1 at the readout frequency. When instead the amplitude of mode 2 is high (for 01, 10, and 11), mode 1 shifts to a higher frequency and the logical output is 0.

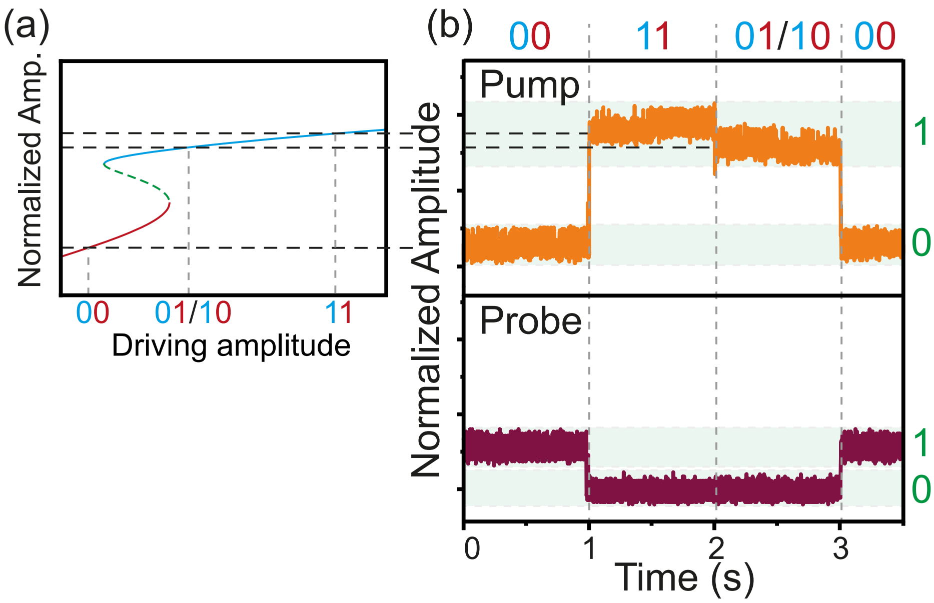

FIG. 1: (a) Simulated amplitude response curve of a Duffing oscillator, displaying its bistable regime of motion. Dashed black lines highlight the driving amplitudes needed to created OR and NOR gates. (b) Top panel: response amplitude of mode 2 as a function of time, for the four combinations of two logical inputs, as indicated by the numbers on top. Bottom panel: response amplitude of mode 1 as a function of time, for the same logical inputs.

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