Time-resolved nonlinear coupling between orthogonal flexural modes of a pristine GaAs nanowire - Supplementary Information

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I. EQUATIONS OF MOTION

To derive the equations of motion for the two modes we treat the nanowire as an isotropic inextensible singly clamped Euler-Bernoulli beam. Following the approach used by Crespo da Silva and Glynn\(^1,2\) we obtain two nonlinear equations describing the flexural vibrations of the beam, for the displacement \(\tilde{u}\):

\[
\begin{align*}
    m\dddot{\tilde{u}} + \tilde{\eta}_1\dot{\tilde{u}} + D_1\dddot{\tilde{w}} + D_1 \left( \dddot{\tilde{u}} \left( \tilde{\dot{u}} \tilde{\dddot{u}} + \tilde{\dddot{u}} \tilde{\dot{u}} \right) \right) ' \\
    + \left( (1 - D_2) \right) \left[ \dddot{\tilde{u}} \int_0^s \tilde{w}^' \tilde{w}^' \, ds - \dddot{\tilde{w}} \int_0^s \tilde{u}^' \tilde{w}^' \, ds \right] \\
    - \frac{(1 - D_2)^2}{D_k} \left[ \dddot{\tilde{u}} \int_0^s \int_0^s \tilde{w}^' \tilde{w}^' \, ds \, ds \right] ' \\
    + \frac{1}{2} m \left[ \tilde{u}^' \int_0^s \int_0^s \dddot{\tilde{u}}^2 + \dddot{\tilde{w}}^2 \, ds \, ds \right] ' = \tilde{Q}_1 - \left( \dddot{\tilde{u}} \int_0^s \tilde{Q}_2 \, ds \right) ' 
\end{align*}
\]

and a symmetric one for the displacement \(\tilde{w}\). Here the dots and primes stand for derivative in time \(\tilde{t}\) and in arc length \(s\) respectively, \(m\) is the mass per unit length (\(= \rho d_1 d_2\) with \(d_1\) and \(d_2\) dimensions of the cross section and \(\rho\) the density), \(\tilde{\eta}_1,2\) the damping coefficient, \(D_1,2\) the bending, and \(D_k\) the torsional stiffnesses of the beam, and \(\tilde{Q}_1,2\) the generalized forces along the two directions. We define \(\tilde{F}_1 = \tilde{Q}_1 - \left( \dddot{\tilde{u}} \int_0^s \tilde{Q}_2 \, ds \right) '\).

Eq. 1 can be rewritten in a dimensionless form\(^3\) substituting \(u = \tilde{u}/d_1, w = \tilde{w}/d_1, x = s/L, \eta_1 = \tilde{\eta}_1 L^4/(D_1 \tau)\) and scaling time with \(\tau = L^2 \sqrt{m/D_1}\).

Applying the Galerkin method for the first mode in the two directions \(u(x, t) = a(t)\xi(x)\) and \(w(x, t) = b(t)\xi(x)\) with \(\xi(x)\) the first flexural mode shape, equal in both directions, we obtain:

\[
\begin{align*}
    \dddot{a} + \omega_1^2 a + \eta_1 \dot{a} + \alpha \left( \frac{d_1}{L} \right)^2 a^3 + \left( \alpha + \frac{1 - D_2}{D_1} \beta_1 - D_1 \frac{1 - D_2}{D_k} \beta_2 \right) \left( \frac{d_1}{L} \right)^2 a b^2 \\
    + \gamma \left( \frac{d_1}{L} \right)^2 a (\dddot{a} + \dot{a} + \dot{b}^2 + \dddot{b}) = \varepsilon F_1 
\end{align*}
\]
where $F_1$ is the scaled dimensionless version of $\tilde{F}_1$, and

$$\alpha = \int_0^1 \xi(x) \left( \xi'(x) \left( \xi' (x) \xi''(x) \right) \right)' \, dx = 40.41$$

$$\beta_1 = \left[ \int_0^1 \xi(x) \left( \xi'' (x) \int_1^x \left( \xi'' (x) \right)^2 \, dx_1 \right) \right]' \, dx - \left[ \int_0^1 \xi(x) \left( \xi'' (x) \int_1^x \xi'(x) \, dx_1 \right) \right]' \, dx = -20.11$$

$$\beta_2 = \int_0^1 \xi(x) \left( \left( \xi'' (x) \int_0^x \int_1^x \xi'' (x) \, dx_2 \, dx_1 \right) \right)' \, dx = 16.60$$

$$\gamma = \int_0^1 \xi(x) \left( \xi'(x) \int_0^x \int_1^x \xi'(x_2) \, dx_2 \, dx_1 \right)' \, dx = 4.60$$

$$\varepsilon = \int_0^1 \xi(x) \, dx = 0.78$$

From our COMSOL simulations, the $\sim 0.5\%$ difference in the cross section of the nanowire is already enough to produce a frequency splitting of the two perpendicular modes similar to what is observed, such that $D_1/D_2 \approx 1.01$. We also consider the beam to have high torsional rigidity compared to the flexural rigidity, such that $D_k \gg D_{1,2}$. Finally, we find that nonlinear damping is negligible by evaluating the critical frequency at which the bistability of the Duffing regime starts to occur. As a result, we obtain the final simplified equation of motion in one direction:

$$\ddot{a} + \omega_1^2 a + \eta_1 \dot{a} + \alpha \left( \frac{d_1}{L} \right)^2 a^3 + \alpha \left( \frac{d_1}{L} \right)^2 a b^2 = \varepsilon F_1$$

Note that, from equation 1 the coupling coefficient with dimensions would be defined as:

$$\tilde{\alpha}_1 = \frac{D_1}{mL} \int_0^L \xi(s) \left( \xi'(s) \left( \xi(s) \xi''(s) \right) \right)' \, ds$$

**II. MECHANICAL LOGIC**

Due to the Duffing nonlinearity, when sweeping the driving amplitude at fixed frequency $f_2$, we observe a high jump between two levels in the response amplitude of mode 2, at a critical driving amplitude. These two levels in the response are used to encode logical 0 and 1 output states$^{4,5}$. The two inputs correspond to two signal voltages which are summed and subsequently applied to the driving PZT. Logical 0 and 1 input states are defined by low
and high driving voltages, respectively (See Fig. 1a). As shown in Fig. 1b (upper panel), we obtain a high response when one or both inputs are high (01, 10, or 11) and a low response when both input signals are low (00). This is therefore a realization of a logical OR gate. This OR gate is converted into a NOR gate by taking as output the response amplitude of mode 1 at $f_1$ (lower panel Fig. 1b). When mode 2 is at the low level (for input 00) there is almost no interaction between the two modes and we have the maximum response (logical 1) of mode 1 at the readout frequency. When instead the amplitude of mode 2 is high (for 01, 10, and 11), mode 1 shifts to a higher frequency and the logical output is 0.

FIG. 1: (a) Simulated amplitude response curve of a Duffing oscillator, displaying its bistable regime of motion. Dashed black lines highlight the driving amplitudes needed to created OR and NOR gates. (b) Top panel: response amplitude of mode 2 as a function of time, for the four combinations of two logical inputs, as indicated by the numbers on top. Bottom panel: response amplitude of mode 1 as a function of time, for the same logical inputs.

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