Class # 2

①

Gray Code

Used for mechanical shaft encoders, etc. Its main property is that only one bit changes in going from one state to the next. This is a sort of error prevention strategy as we will see. This is especially useful when one cannot garantee that all bit change simultaneously. Example in slides ...

Combinatorial vs Sequential Logic

Combinatorial

adding numbers
comparing numbers
parity check
convert binary to decimal

Output is predetermined function of input.

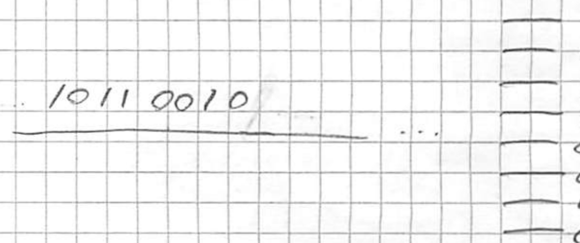
Can be expressed by Boolean algebra and performed with gates.

Sequential

serial bits ⟶ parallel bits

counting the number of 1's in a sequence

giving an output every four input pulses

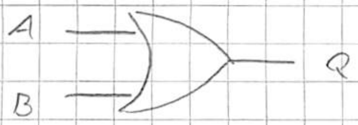


Output is based on the history of the input.

⟶ Require digital memory implemented by flip-flops.

Gates

OR



A	B	Q
0	0	0
0	1	1
1	0	1
1	1	1

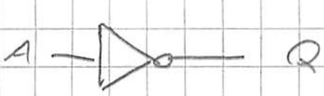
$A + B$, A OR B

AND

A	B	Q
0	0	0
0	1	0
1	0	0
1	1	1

$A \cdot B$, A AND B , AB

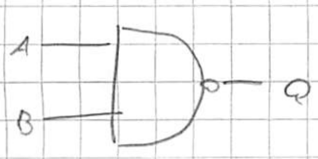
INVERT



A	Q
0	1
1	0

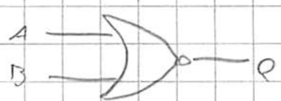
$\bar{A}$, NOT A , A^*

NAND



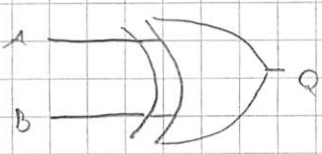
A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0

NOR



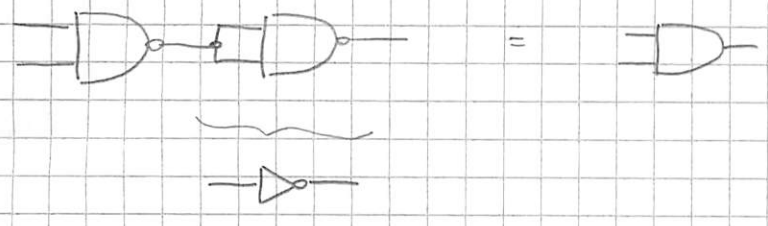
A	B	Q
0	0	1
0	1	0
1	0	0
1	1	0

XOR



A	B	Q
0	0	0
0	1	1
1	0	1
1	1	0

Gate Interchangability

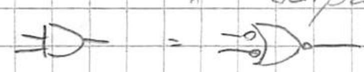


In general → multiple use of an inverting gate (NAND or NOR) makes any function

← universal gates

De Morgan's Theorem

You can swap shapes if you invert all inputs & outputs:



0	0	0
0	1	0
1	0	0
1	1	1

→

T	T	T
T	F	T
F	T	T
F	F	F

→

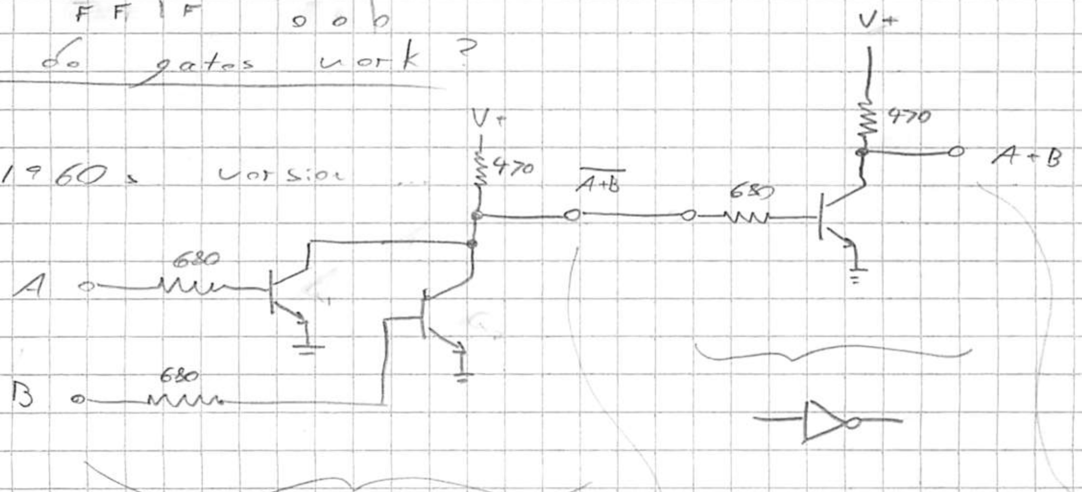
1	1	1
1	0	1
0	1	1
0	0	0

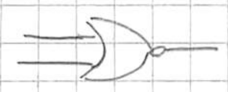
e.g.

, etc.

How do gates work?

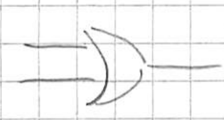
1960s version ...



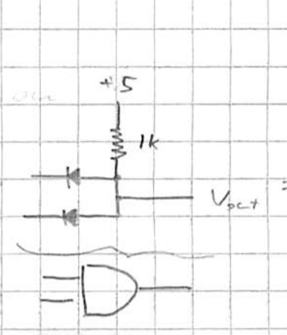


A	B	O
0	0	1
0	1	0
1	0	0
1	1	0

AB	O
00	0
01	1
10	1
11	1



Slow!



Combinatorial Logic

$$A \oplus B = \bar{A}B + A\bar{B}$$

w/ identities ...

$$A \oplus B = A\bar{A} + A\bar{B} + B\bar{A} + B\bar{B}$$
$$= A(\bar{A} + \bar{B}) + B(\bar{A} + \bar{B})$$
$$= A(\overline{AB}) + B(\overline{AB})$$

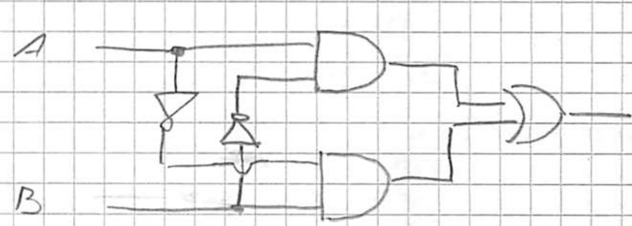
w/ DeMorgan's Theorem!

$$A \oplus B = (A + B)(\overline{AB})$$

XOR

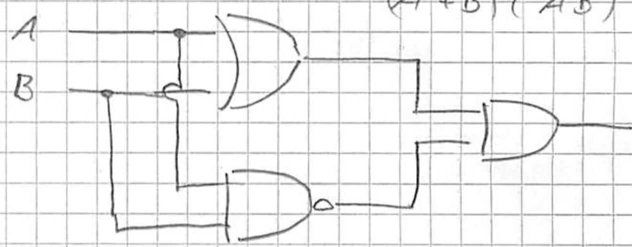
A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

$\therefore$ XOR $\quad \bar{A}B + A\bar{B}$



or

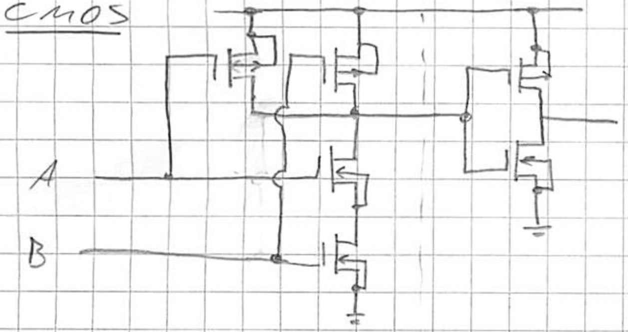
$(A+B)(\overline{AB})$

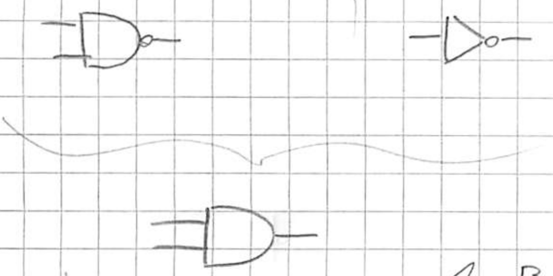


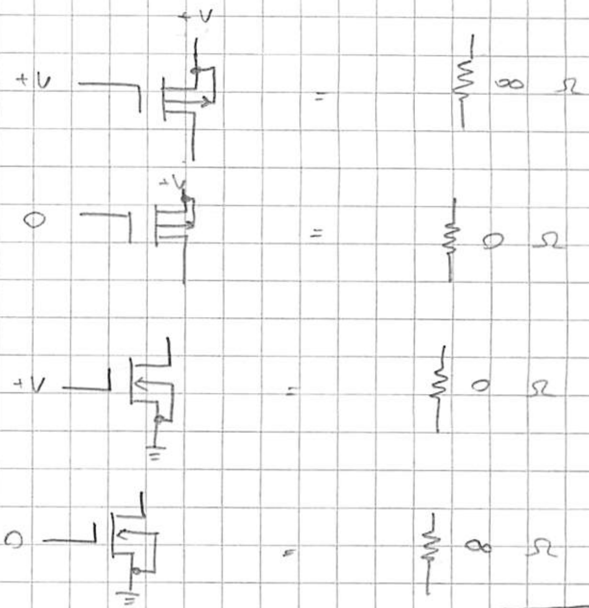
Karnaugh map slide ...

(Complementary Metal Oxide Semiconductor)

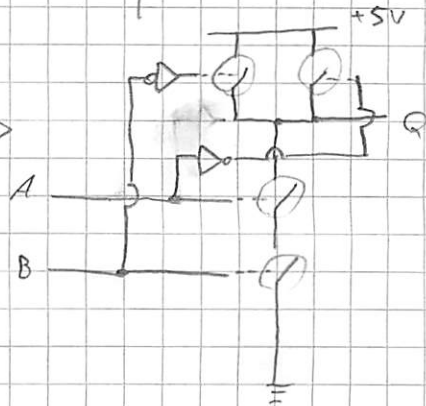
CMOS





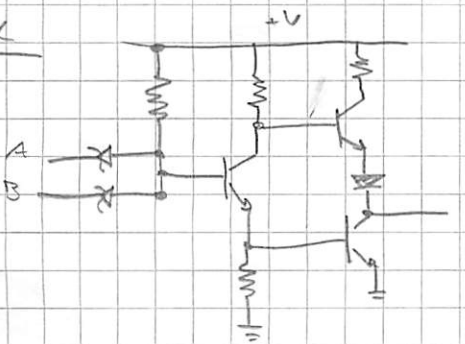


A	B	O
0	0	0
0	1	0
1	0	0
1	1	1

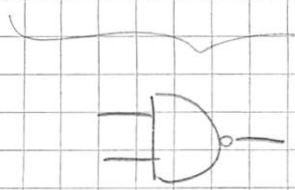


(Transistor - Transistor Logic)

TTL

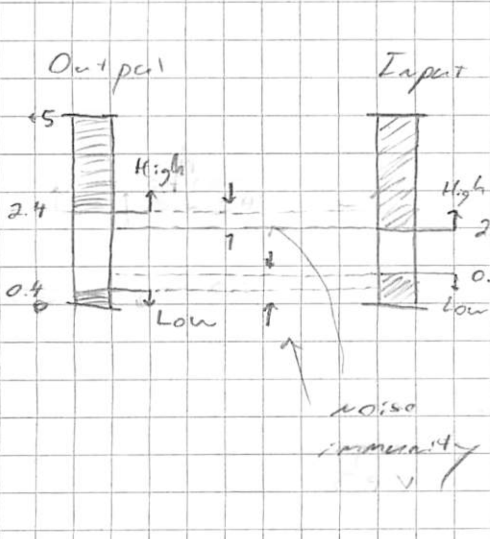


A	B	O
0	0	1
0	1	1
0	1	1
1	1	0

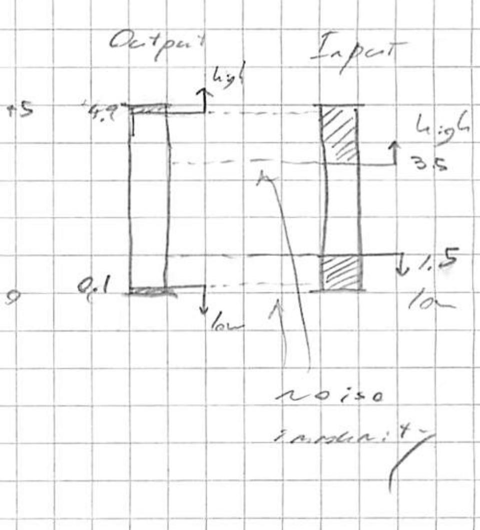


Thresholds & Noise Margins

TTL

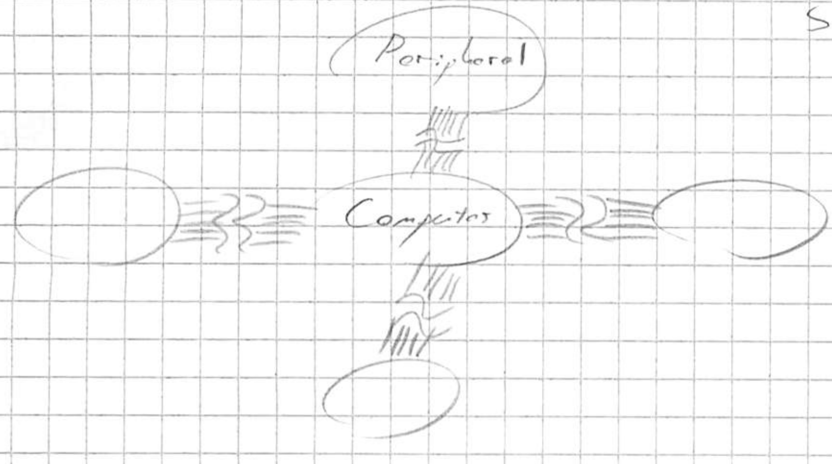


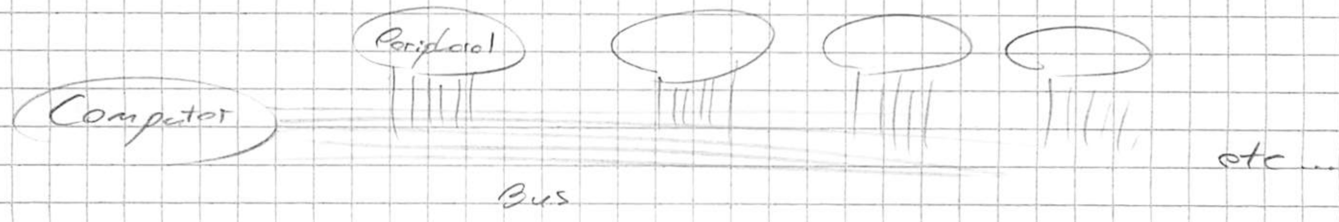
CMOS



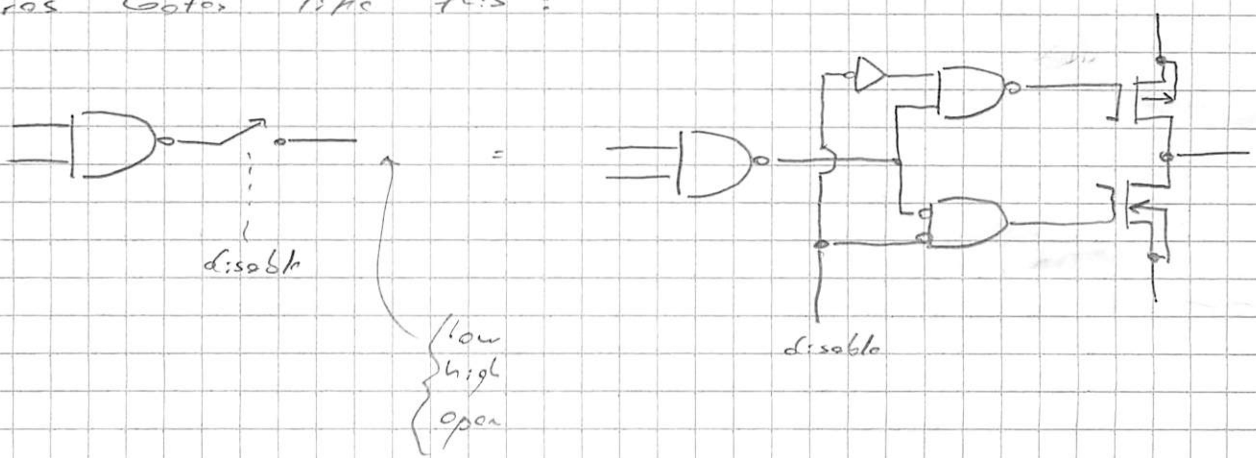
3-state Logic

Show Slide





Requires Gates like this:



Multiplexers

For routing signals ...

8-input MUX :