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## Quantum theory of optomechanical cooling

Florian Marquardt<sup>a\*</sup>, A.A. Clerk<sup>b</sup> and S.M. Girvin<sup>c</sup>

<sup>a</sup>*Department of Physics, Center for NanoScience and Arnold-Sommerfeld Center for Theoretical Physics, Ludwig-Maximilians-Universität München, München, Germany;*  
<sup>b</sup>*Department of Physics, McGill University, Montreal, QC, Canada;* <sup>c</sup>*Department of Physics, Yale University, New Haven, CT, USA*

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We review the quantum theory of cooling of a mechanical oscillator subject to the radiation pressure force due to light circulating inside a driven optical cavity. Such optomechanical setups have been used recently in a series of experiments by various groups to cool mechanical oscillators (such as cantilevers) by factors reaching  $10^5$ , and they may soon go to the ground state of mechanical motion. We emphasize the importance of the sideband-resolved regime for ground state cooling, where the cavity ring-down rate is smaller than the mechanical frequency. Moreover, we illustrate the strong coupling regime, where the cooling rate exceeds the cavity ring-down rate and where the driven cavity resonance and the mechanical oscillation hybridize.

**Keywords:** cavity QED; optomechanics; micromechanics; sideband cooling; radiation pressure

### 1. Introduction

The interaction of light with matter has been at the heart of the development of quantum mechanics since its inception. As for the mechanical effects of light, these become most pronounced in a setup where the light intensity is resonantly enhanced (i.e. an optical cavity) and where photons transfer maximum momentum to a mechanical object, e.g. by being reflected multiple times from a movable mirror attached to a cantilever. The study of radiation pressure effects on a movable mirror was pioneered in seminal papers by Braginsky and co-workers [1,2]. Strong changes of the mechanical properties of the mirror were observed later in an experiment by the Walther group [3], where a macroscopic mirror was found to exhibit two stable equilibrium positions under the action of the cavity's radiation field. The most recent series of activities in this field started with experiments observing optomechanical cooling first using feedback [4,5] and later [6–12] using the intrinsic effect discussed in the following, i.e. 'passive' cooling, without active feedback, using purely the radiation pressure backaction. In addition, we note the self-induced optomechanical oscillations [13] that have been observed in radiation-pressure driven microtoroidal optical resonators [14,15] and other setups [16,17]. For a recent review see [18]. The study of these systems has been made even more fruitful by the

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\*Corresponding author. Email: [florian.marquardt@physik.lmu.de](mailto:florian.marquardt@physik.lmu.de)

realization that the same (or essentially similar) physics may be observed in systems ranging from driven LC circuits coupled to cantilevers [19] over superconducting single electron transistors and microwave cavities coupled to nanobeams [20–26] to clouds of cold atoms in an optical lattice, whose oscillations couple to the light field [27,28]. Cooling to the ground-state may open the door to various quantum effects in these systems, including ‘cat’ states [29], entanglement [30,31], quantum nonlinear dynamics [32], and Fock state detection [12]. Therefore, optomechanical ground-state cooling is currently probably the highest priority in the field.

All the intrinsic optomechanical cooling experiments are based on the fact that the radiation field introduces extra damping for the cantilever. In such a classical picture, the extra damping reduces the effective temperature of the single mechanical mode of interest, while leaving the bulk temperature of the cantilever the same (in the absence of appreciable light absorption). The resulting effective temperature is related to the bath temperature  $T$  by  $T_{\text{eff}}/T = \Gamma_{\text{M}}/(\Gamma_{\text{opt}} + \Gamma_{\text{M}})$ , where  $\Gamma_{\text{M}}$  and  $\Gamma_{\text{opt}}$  are the intrinsic mechanical damping rate and the optomechanical cooling rate, respectively. Thus, there is no limit to cooling in this regime, provided the laser power (and thus the cooling rate  $\Gamma_{\text{opt}}$ ) can be increased without any deleterious effects such as unwanted heating by absorption, and provided the cooling rate remains sufficiently smaller than the mechanical frequency and the cavity ring-down rate. However, at sufficiently low temperatures, the unavoidable photon shot noise inside the cavity counteracts cooling. To study the resulting quantum limits to cooling, a fully quantum-mechanical theory is called for, which we provided in [33], based on the general quantum noise approach. Independently, a derivation emphasizing the analogy to ion sideband cooling was developed in [34]. In the present paper, we will review and illustrate our theory. We start by outlining the basic classical picture, then present the quantum noise approach that provides a transparent and straightforward way to derive cooling rates and quantum limits for the phonon number. Finally, we illustrate the strong coupling regime that was first predicted in [33].

## 2. Basic classical picture

In this section, we briefly review the basic classical description of optomechanical cooling. This was pioneered by Braginsky and co-workers in their work on optomechanical damping [1,2], while the associated cooling effect was first demonstrated only rather recently [7].

The main ingredient for optomechanical cooling is the appearance of extra damping, ideally without extra fluctuations. This damping is introduced because the light-induced force reacts with a finite delay time. In the case of radiation pressure, this comes about due to the ring-down time  $\kappa^{-1}$  of the cavity. On the other hand, bolometric (i.e. photothermal) forces are produced when a bimorph cantilever absorbs some of the radiation circulating inside the cavity. When bolometric forces dominate, it is the finite time of thermal conductance that sets the time-lag between the impinging radiation intensity and the resulting change in the cantilever temperature, which is proportional to the force.

The physical picture behind damping is simplest when the time-lag is small compared to the oscillation period of the cantilever. Then both radiation pressure and bolometric forces give rise to the same physics, modulo the appearance of a different time-scale in the two cases.

To describe this physics, we first fix our coordinate system: increasing the displacement  $x$  of the cantilever means elongating the cavity, and thus the optical resonance frequency (of the mode of interest) decreases. Let us consider the cantilever being placed at some location to the left of the resonance (Figure 1). This means that the optical mode frequency is still higher than the frequency of the incoming laser radiation, which is therefore red-detuned with respect to the optical resonance. Now imagine moving the cantilever in a small cycle. As it moves towards the resonance with a finite speed, the light-induced force does work on the cantilever. However, due to the time-lag it remains smaller than it would be in the case of infinitely slow (adiabatic) motion. Conversely, as the cantilever moves back again in the second half of the cycle, the force extracts energy and it is larger than for adiabatic motion. In total, the work done during such a cycle by the light-induced force is negative, i.e. mechanical energy is extracted from the cantilever (Figure 1(b)). This kind of physics may be modeled by writing down a simple relaxation-type equation for the force, which tries to reach its proper  $x$ -dependent value  $\mathcal{F}(x)$  with some time-lag:

$$\dot{F}(t) = \frac{\mathcal{F}(x(t)) - F(t)}{\tau} \tag{1}$$

The cantilever is a damped harmonic oscillator driven by the light-induced force:

$$m\ddot{x} = -m\omega_M^2(x - x_0) - m\Gamma_M\dot{x} + F, \tag{2}$$

where  $x_0$  is the mechanical equilibrium position in the absence of radiation pressure,  $m$  is the cantilever mass,  $\omega_M$  is its mechanical frequency, and  $\Gamma_M$  is the intrinsic damping rate. Linearizing Equation (1) with respect to small displacements from the mechanical equilibrium position  $\bar{x}$  (calculated in the presence of radiation pressure) and inserting it into the equation of motion of the cantilever then yields the extra damping force. In Fourier space, where  $x(t) = \int x[\omega]\exp(-i\omega t)d\omega/2$ , we find the following linearized equation of motion (at  $\omega \neq 0$ ):

$$-\omega^2 mx[\omega] = -m\omega_M^2 x[\omega] + im\omega\Gamma_M x[\omega] + \mathcal{F}'(\bar{x})x[\omega]/(1 - i\omega\tau). \tag{3}$$

Comparing the last two terms (the intrinsic damping with the imaginary part of the optomechanical term), we find that the optomechanical damping rate is given by

$$\Gamma_{\text{opt}} = \frac{\mathcal{F}'(\bar{x})}{m\omega_M} \frac{\omega_M\tau}{1 + (\omega_M\tau)^2}, \tag{4}$$

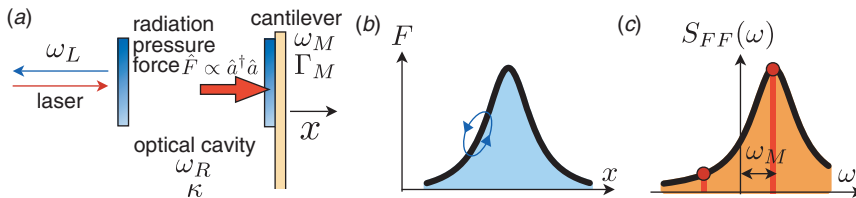


Figure 1. (a) The standard optomechanical setup treated in the text: a driven optical cavity with a movable mirror. (b) Moving the mirror in a cycle can result in work extracted by the light-field, due to the finite cavity ring-down rate. (c) Asymmetric quantum noise spectrum for the radiation pressure force noise, according to Equation (14). (The color version of this figure is included in the online version of the journal.)

for the simple ansatz of Equation (1). According to this analysis, one would expect the maximum effect to occur when  $\omega_M \tau = 1$ , i.e. when the time-delay matches the period of the cantilever motion. As we will see further below, this conclusion is not upheld by the full quantum-mechanical analysis for the case of radiation pressure.

Equation (1) and the subsequent analysis holds exactly for the bolometric force. In that case,  $\tau$  is the finite time of thermal conductance and  $\mathcal{F}(x) = \mathcal{F}_{\max} I(x) / I_{\max}$  is the displacement-dependent bolometric force, where  $I(x) = I_{\max} / (1 + (2\Delta(x)/\kappa)^2)$  is the intensity profile and  $\Delta(x) = x\omega_R/L$  is the position-dependent detuning between incoming laser radiation and optical resonance at  $x=0$ . For the radiation-pressure force, we can use the present analysis only in the regime of  $\kappa \gg \omega_M$ , and only if we allow for a position dependent relaxation rate  $1/\tau$  (see [33]).

In both cases, however, the shape of  $\Gamma_{\text{opt}}$  as a function of cantilever position  $\bar{x}$  is determined by the slope of the intensity profile, i.e. in particular by the sign of  $\mathcal{F}'$ . To the left of the resonance, where  $\mathcal{F}' > 0$ , we indeed obtain extra damping:  $\Gamma_{\text{opt}} > 0$ . As long as there are no extra fluctuations introduced by the light-induced force (i.e. if we may disregard shot noise), the effective temperature of the mechanical degree of freedom is therefore reduced according to the ratio of intrinsic and optomechanical damping rates:

$$T_{\text{eff}} = T \frac{\Gamma_M}{\Gamma_M + \Gamma_{\text{opt}}}. \quad (5)$$

This can be obtained, for example, by solving the Langevin equation that includes the thermal fluctuations of the mechanical heat bath (whose strength is set by  $\Gamma_M$  according to the fluctuation-dissipation theorem). Then the effective temperature may be defined according to the equipartition theorem:  $m\tilde{\omega}_M^2 \langle (x - \bar{x})^2 \rangle = k_B T_{\text{eff}}$  where  $\tilde{\omega}_M$  contains the frequency renormalization due to the real part of the optomechanical term in Equation (3).

### 3. Quantum noise approach

The quantum regime is reached once the simple classical relation for the effective temperature, Equation (5), ceases to be valid. This happens when one has to take into account the shot noise that tends to heat the cantilever motion. It enforces a lower bound for the cantilever phonon number that can be reached by optomechanical cooling, i.e. a finite quantum limit, which we are going to calculate. Crucially, we will also show that a proper choice of parameters can make this bound on the phonon number become much smaller than one, indicating that ground state cooling is possible.

The quantum picture can also be understood in terms of Raman scattering: incoming photons, red-detuned with respect to the optical resonance, absorb a phonon from the cantilever, thereby cooling it. However, there is also a finite probability for phonon emission, and thus heating. The purpose of a quantum theory is to discuss the balance of these effects.

The idea behind the quantum noise approach to quantum-dissipative systems is to describe the environment fully by the correlator of the fluctuating force that couples to the quantum system of interest. If the coupling is weak enough, knowledge of the correlator is

sufficient to fully describe the influence of the environment. Essentially, the results of this approach are based on leading-order perturbation theory, i.e. Fermi's Golden Rule and related expressions. It yields rates of second order in the coupling that can be used inside standard master equations [35], to calculate the steady-state occupations and other quantities of interest. In our case, this means looking at the spectrum of the radiation pressure force fluctuations, which are produced by the shot noise of photons inside the driven optical cavity mode, i.e. a nonequilibrium environment. In other applications, we might be dealing with the electrical field fluctuations produced, e.g. by a driven electronic circuit (superconducting single electron transistor, quantum point contact, LC circuit) capacitively coupled to some nanobeam. The general formulas remain the same for all of these cases, and only the noise spectrum changes.

The Fourier transform of the force correlator defines the spectral noise density:

$$S_{FF}(\omega) = \int dt \exp(i\omega t) \langle \hat{F}(t) \hat{F}(0) \rangle. \quad (6)$$

The noise spectrum  $S_{FF}$  is real-valued and non-negative. However, in contrast to the classical case, it is asymmetric in frequency, since  $\hat{F}(t)$  and  $\hat{F}(0)$  do not commute. This asymmetry has an important physical meaning: contributions at positive frequencies indicate the possibility of the environment to absorb energy, while those at negative frequencies imply its ability to release energy (to the cantilever). All the optomechanical effects can be described in terms of  $S_{FF}$ , as long as the coupling is weak.

The optomechanical damping rate is given by the difference of noise spectra at positive and negative frequencies,

$$\Gamma_{\text{opt}} = \frac{x_{\text{ZPF}}^2}{\hbar^2} [S_{FF}(\omega_M) - S_{FF}(-\omega_M)]. \quad (7)$$

Here  $x_{\text{ZPF}} = [\hbar/(2m\omega_M)]^{1/2}$  is the zero-point amplitude of mechanical motion. This formula is obtained by applying Fermi's Golden Rule to derive the transition rates arising from the coupling of the cantilever to the light field, i.e. from the term  $\hat{H}_{\text{int}} = -\hat{F}\hat{x}$  in the Hamiltonian. These are

$$\Gamma_{\downarrow}^{\text{opt}} = \frac{x_{\text{ZPF}}^2}{\hbar^2} S_{FF}(\omega_M), \quad \Gamma_{\uparrow}^{\text{opt}} = \frac{x_{\text{ZPF}}^2}{\hbar^2} S_{FF}(-\omega_M). \quad (8)$$

These rates enter the complete master equation for the density matrix  $\hat{\rho}$  of the cantilever in the presence of the equilibrium heat bath (that would lead to a thermal population  $\bar{n}_{\text{th}}$ ) and the radiation field:

$$\dot{\hat{\rho}} = \left[ (\Gamma_{\downarrow}^{\text{opt}} + \Gamma_M(\bar{n}_{\text{th}} + 1))\mathcal{D}[\hat{a}] + (\Gamma_{\uparrow}^{\text{opt}} + \Gamma_M\bar{n}_{\text{th}})\mathcal{D}[\hat{a}^\dagger] \right] \hat{\rho}. \quad (9)$$

Here the equation has been written in the interaction picture (disregarding the oscillations at  $\tilde{\omega}_M$ ), and

$$\mathcal{D}[\hat{A}]\hat{\rho} = \frac{1}{2}(2\hat{A}\hat{\rho}\hat{A}^\dagger - \hat{A}^\dagger\hat{A}\hat{\rho} - \hat{\rho}\hat{A}^\dagger\hat{A}) \quad (10)$$

is the standard Lindblad operator for downward ( $\hat{A} = \hat{a}$ ) or upward ( $\hat{A} = \hat{a}^\dagger$ ) transitions in the oscillator. Restricting ourselves to the populations  $\rho_{mm}$ , we obtain the equation for the phonon number  $\bar{n} = \langle \hat{n} \rangle = \sum_n n \rho_{nn}$ :

$$\dot{\bar{n}} = \Gamma_M \bar{n}_{\text{th}} + \Gamma_{\uparrow}^{\text{opt}} - (\Gamma_M + \Gamma_{\text{opt}}) \bar{n}, \quad (11)$$

which yields the steady-state phonon number in the presence of optomechanical cooling:

$$\bar{n}_M = \frac{\Gamma_M \bar{n}_{\text{th}} + \Gamma_{\text{opt}} \bar{n}_M^{\text{O}}}{\Gamma_M + \Gamma_{\text{opt}}}. \quad (12)$$

This is the weighted average of the thermal and the optomechanical phonon numbers. It represents the correct generalization of the classical formula for the effective temperature, Equation (5). Here

$$\bar{n}_M^{\text{O}} = \frac{\Gamma_{\uparrow}^{\text{opt}}}{\Gamma_{\text{opt}}} = \frac{1}{\Gamma_{\downarrow}^{\text{opt}}/\Gamma_{\uparrow}^{\text{opt}} - 1} = \left[ \frac{S_{FF}(\omega_M)}{S_{FF}(-\omega_M)} - 1 \right]^{-1} \quad (13)$$

is the minimal phonon number reachable by optomechanical cooling. This quantum limit is reached when  $\Gamma_{\text{opt}} \gg \Gamma_M$ . Then, the cooling effect due to extra damping is balanced by the shot noise in the cavity, which leads to heating.

The radiation pressure force is proportional to the photon number:  $\hat{F} = (\hbar\omega_R/L)\hat{a}^\dagger\hat{a}$ . A brief calculation for the photon number correlator inside a driven cavity [33] yields its spectrum in the form of a Lorentzian that is shifted by the detuning  $\Delta = \omega_L - \omega_R$  between laser and optical resonance frequency  $\omega_R$ :

$$S_{FF}(\omega) = \left( \frac{\hbar\omega_R}{L} \right)^2 \bar{n}_p \frac{\kappa}{(\omega + \Delta)^2 + (\kappa/2)^2}, \quad (14)$$

where  $\bar{n}_p$  is the photon number circulating inside the cavity. This asymmetric quantum noise spectrum is illustrated in Figure 1(c), while a plot of the resulting steady-state phonon number is shown in Figure 2(a).

Inserting this spectrum into Equations (7) and (13) yields the optomechanical cooling rate and the minimum phonon number as a function of detuning  $\Delta$ . The minimum of  $\bar{n}_M^{\text{O}}$  is reached at a detuning  $\Delta = -[\omega_M^2 + (\kappa/2)^2]^{1/2}$ , and it is (see Figure 2(b)):

$$\min \bar{n}_M^{\text{O}} = \frac{1}{2} \left( \left[ 1 + \left( \frac{\kappa}{2\omega_M} \right)^2 \right]^{1/2} - 1 \right). \quad (15)$$

For slow cantilevers,  $\omega_M \ll \kappa$ , we have  $\min \bar{n}_M^{\text{O}} = \kappa/(4\omega_M) \gg 1$ . Ground-state cooling becomes possible for high-frequency cantilevers (and/or high-finesse cavities), when  $\kappa \ll \omega_M$ . Then, we find

$$\min \bar{n}_M^{\text{O}} \approx \left( \frac{\kappa}{4\omega_M} \right)^2. \quad (16)$$

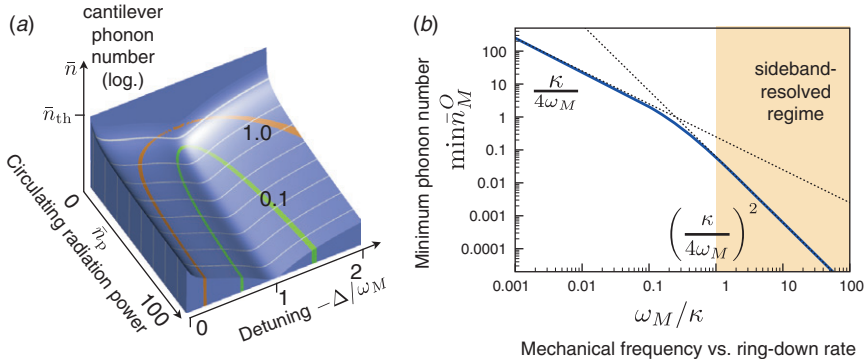


Figure 2. (a) Cantilever phonon number as a function of circulating radiation power inside the cavity and as a function of detuning between laser and optical resonance. The mean phonon number in steady state is plotted on a logarithmic scale. Contour lines indicate  $\bar{n} = 1$  and  $\bar{n} = 0.1$ . For this plot, the following parameters have been used:  $\omega_M/\kappa = 0.3$ ,  $\bar{n}_{\text{th}} = 10^3$ ,  $Q = \omega_M/\Gamma_M = 10^6$ , and  $(\omega_R/\omega_M)(x_{\text{ZPF}}/L) \approx 0.012$ . (b) The minimum phonon number as a function of the ratio between the mechanical frequency  $\omega_M$  and the optical cavity's ring-down rate  $\kappa$ , according to Equation (15). Ground-state cooling is possible in the regime  $\omega_M \gg \kappa$ , i.e. the 'good cavity' or 'resolved sideband' limit. (The color version of this figure is included in the online version of the journal.)

As explained in [34], these two regimes can be brought directly into correspondence with the known regimes for laser-cooling of harmonically bound atoms, namely the Doppler limit for  $\omega_M \ll \kappa$  and the resolved sideband regime for  $\omega_M \gg \kappa$ . In the resolved sideband limit the Stokes and anti-Stokes lines reflected from the cantilever–cavity setup, which are at  $\omega_L \pm \omega_M$ , can be resolved from the main line at  $\omega_L$ . Only in this limit is ground state cooling possible.

#### 4. Strong coupling effects

Up to now, we have assumed that the coupling between light and mechanical degree of freedom is sufficiently weak to allow for a solution in terms of a master equation, employing the rates obtained from the quantum noise approach. However, as the coupling becomes stronger (e.g. by increasing the laser input power),  $\Gamma_{\text{opt}}$  may reach the cavity decay rate  $\kappa$ . Then, the spectrum of force fluctuations is itself modified by the presence of the cantilever. It becomes necessary to solve for the coupled dynamics of the light field and the mechanical motion. This has been done in [33], by writing down the Heisenberg equations of motion for the cantilever and the optical mode, and solving them after linearization. Here we will only discuss the result.

To analyze these features, let us consider the spectrum of the cantilever motion,

$$S_{cc}(\omega) = \int dt \exp(i\omega t) \langle \hat{c}^\dagger(t) \hat{c}(0) \rangle, \quad (17)$$

where  $\hat{c}$  is the annihilation operator for the cantilever harmonic oscillator. At weak coupling, this spectrum displays a peak at the (renormalized) cantilever frequency, i.e. at  $\omega = -\tilde{\omega}_M$  (the minus sign is a consequence of our choice of the definition, following [33]).



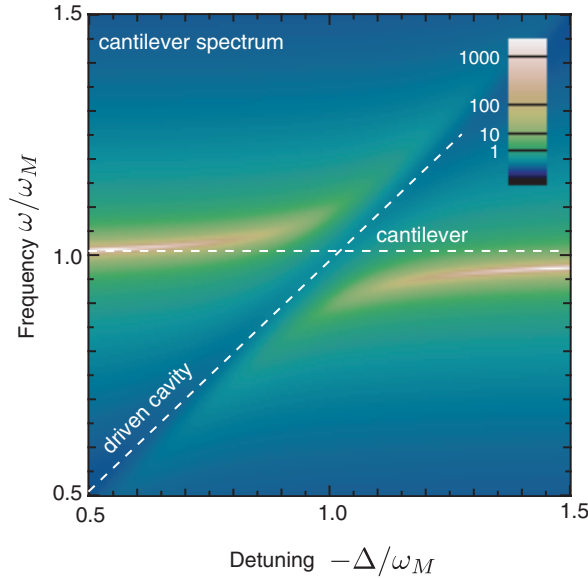


Figure 3. The spectrum  $S_{cc}(-\omega)$  of the cantilever motion, as a function of detuning  $\Delta$  and spectral frequency  $\omega$ , for fixed circulating power. For the plot shown here, the strong-coupling regime is reached, i.e.  $\Gamma_{\text{opt}}/\kappa$  becomes larger than one near resonance. Consequently, the driven cavity resonance and the cantilever resonance hybridize, and an avoided crossing is observed. The parameters for this plot are:  $\kappa/\omega_M = 0.1$ ,  $\Gamma_M/\omega_M = 10^{-5}$ ,  $(\omega_R/\omega_M)(x_{\text{ZPF}}/L) = 0.01$ ,  $\bar{n}_p = 100$ , and  $\bar{n}_{\text{th}} = 10^3$ , and the color scale indicates the values of  $\omega_M S_{cc}(-\omega)$ . (The color version of this figure is included in the online version of the journal.)

Its width (FWHM) is given by  $\Gamma_{\text{opt}} + \Gamma_M$ , and its total weight  $\int S_{cc}(\omega) d\omega/2$  yields the phonon number  $\bar{n} = \langle \hat{c}^\dagger \hat{c} \rangle$ . As the laser power is increased, the width increases and the weight diminishes (in the cooling regime).

When  $\Gamma_{\text{opt}}/\kappa$  is no longer much smaller than one, deviations from the weak-coupling results start to appear [33]. The most dramatic effect is observed when  $\Gamma_{\text{opt}}/\kappa > 1/2$ : the peak splits into two (Figure 3). As it were, in this strong-coupling regime one actually encounters the hybridization of two coupled harmonic oscillators, namely the cantilever and the driven optical mode (with an effective frequency set by the detuning). At resonance, i.e. for  $\Delta = -\omega_M$ , the splitting is set by  $2\alpha$ , where the coupling frequency  $\alpha$  is determined by the circulating laser power, the ratio of mechanical zero-point fluctuations to the cavity length, and by the optical resonance frequency:

$$\alpha = \omega_R \bar{n}_p^{-1/2} \frac{x_{\text{ZPF}}}{L}. \quad (18)$$

## 5. Outlook

During the past year, several new ideas have been introduced into the field of optomechanical cooling. For example, placing a movable membrane in the middle of a standard optical cavity [12] can lead to orders of magnitude better performance, as it

separates the mechanical from the optical elements. Such a setup has been used to cool from 300 K down to 7 mK, and it may ultimately be employed for Fock state detection of mechanical vibrations [12]. Even nanomechanical objects (such as nanowires) might be placed inside the standing light wave [36] and cooled by scattering. Doppler cooling of Bragg mirrors may provide another promising approach [37]. Recent experiments on nanomechanical resonators coupled to on-chip microwave transmission line resonators [26,38] and on trapped cold atoms interacting with a light field [28] point towards promising new avenues in the field of optomechanics (see ‘News and Views’ [39]). Furthermore, sideband-resolved cooling with  $\omega_M/\kappa \sim 20$  has been demonstrated recently [40], paving the way for ground-state cooling when combined with cryogenics [41].

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