nature nanotechnology

SUPPLEMENTARY INFORMATION

DOI: 10.1038/NNANO.2016.189

Vectorial scanning force microscopy using a nanowire sensor

Nicola Rossi, Floris R. Braakman, Davide Cadeddu, Denis Vasyukov,
Gözde Tütüncüoglu, Anna Fontcuberta i Morral, and Martino Poggio

DOI: 10.1038/NNANO.2016.189

I. ZOOMED OUT SCANNING ELECTRON MICROGRAPHS OF NANOWIRES AND FINGER GATE SAMPLE

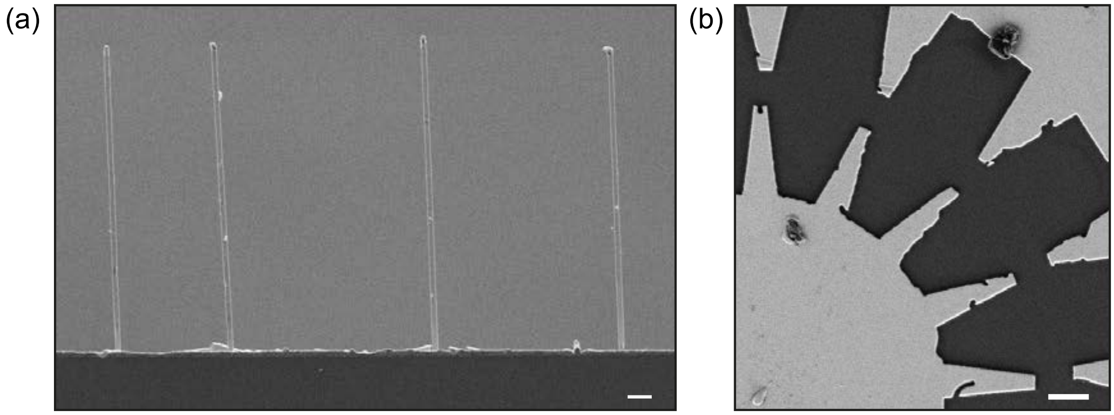

FIG. 1: (a) Zoomed out SEM image of the edge of the NW sample, showing several similar NWs. (b) Zoomed out SEM image of the finger gate sample. The scalebar in both figures corresponds to 2μm.

II. NANOWIRE MOTION IN PRESENCE OF TIP-SAMPLE FORCE

We describe the motion of the NW tip in each of the two fundamental flexural modes as a driven damped harmonic oscillator:

$$m\ddot{r}_i + \Gamma_i \dot{r}_i + k_i r_i = F_{th} + F_i, \tag{S.1}$$

where m is the effective mass of the fundamental flexural modes, F_{th} is the Langevin force, F_i is the component of the tip-sample force along the mode oscillation direction $\hat{\mathbf{r}}_i$, and $i = 1, 2$. Following the treatment of Gloppe et al.[1] and expanding F_i for small oscillations around the equilibrium $r_i = 0$, we have,

$$F_i \approx F_i(0) + r_j \left.\frac{\partial F_i}{\partial r_j}\right|_0 . \tag{S.2}$$

By replacing this expansion in (S.1), we have,

$$m\ddot{r}_i + \Gamma_i \dot{r}_i + k_i r_i = F_{th} + F_i(0) + F_{ij} r_j, \tag{S.3}$$

where we use a shorthand notation for the force derivatives $F_{ij} \equiv \left.\frac{\partial F_i}{\partial r_j}\right|_0$. As (S.3) makes clear, the derivatives of the tip-sample force modify and couple the NW's two unperturbed flexural modes. We now rewrite this equation in vectorial form as,

$$m\ddot{\mathbf{r}} + \bar{\Gamma}\cdot\dot{\mathbf{r}} + \bar{K}\cdot\mathbf{r} = \mathbf{F}_{th} + \mathbf{F}_0, \tag{S.4}$$

where $\mathbf{r} = \binom{r_1}{r_2}$ and the equilibrium tip-sample force $\mathbf{F}_0 = \mathbf{F}(\mathbf{r}=0)$. The dissipation and spring constant matrices are defined by:

$$\bar{\Gamma} \equiv \begin{pmatrix} \Gamma_1 & 0 \\ 0 & \Gamma_2 \end{pmatrix}, \tag{S.5}$$

$$\bar{K} \equiv \begin{pmatrix} k_1 - F_{11} & -F_{21} \\ -F_{12} & k_2 - F_{22} \end{pmatrix}, \tag{S.6}$$

where the role of the shear cross-derivatives, i.e. F_{ij} for $i \neq j$, in coupling the unperturbed NW modes is clear[2]. In the presence of weak tip-surface interactions, as those studied here, the dissipation rates of the NW modes are neglibly small compared to their unperturbed resonant frequencies, i.e. $\frac{\Gamma}{2m} \ll \sqrt{\frac{k}{m}}$. In this limit, the NW mode frequencies and oscillation directions are determined by $\bar{K}$. Therefore, by diagonalizing $\bar{K}$, we find a new pair of uncoupled flexural modes. The corresponding spring constants and mode directions have been modified from the unperturbed state by the spatial tip-sample force derivatives F_{ij}:

$$k_1' = \frac{1}{2}\left[k_1 + k_2 - F_{11} - F_{22} + \sqrt{(k_1 - k_2 - F_{11} + F_{22})^2 + 4F_{12}F_{21}}\right], \tag{S.7}$$

$$\hat{\mathbf{r}}_1' = \frac{1}{\sqrt{(k_2 - F_{22} - k_1')^2 + F_{12}^2}} \begin{pmatrix} k_2 - F_{22} - k_1' \\ F_{12} \end{pmatrix}; \tag{S.8}$$

$$k_2' = \frac{1}{2}\left[k_1 + k_2 - F_{11} - F_{22} - \sqrt{(k_1 - k_2 - F_{11} + F_{22})^2 + 4F_{12}F_{21}}\right], \tag{S.9}$$

$$\hat{\mathbf{r}}_2' = \frac{1}{\sqrt{(k_1 - F_{11} - k_2')^2 + F_{21}^2}} \begin{pmatrix} F_{21} \\ k_1 - F_{11} - k_2' \end{pmatrix}. \tag{S.10}$$

These new modes remain orthogonal ($\hat{\mathbf{r}}_1' \cdot \hat{\mathbf{r}}_2' = 0$) for conservative force fields ($\nabla \times \mathbf{F} = 0$, i.e. $F_{12} - F_{21} = 0$), but lose their orthogonality for non-conservative force fields.

For tip-sample force derivatives that are much smaller than the bare NW spring constants – which is the case here – the modified spring constants and the modified mode directions

DOI: 10.1038/NNANO.2016.189

in (S.7) - (S.10) can be approximated to first order in the derivatives:

$$k_1' \approx k_1 - F_{11}, \tag{S.11}$$

$$\hat{\mathbf{r}}_1' \approx \frac{1}{\sqrt{(k_1-k_2)^2 + F_{12}^2}} \begin{pmatrix} k_1 - k_2 \\ -F_{12} \end{pmatrix}; \tag{S.12}$$

$$k_2' \approx k_2 - F_{22}, \tag{S.13}$$

$$\hat{\mathbf{r}}_2' \approx \frac{1}{\sqrt{(k_1-k_2)^2 + F_{21}^2}} \begin{pmatrix} F_{21} \\ k_1 - k_2 \end{pmatrix}. \tag{S.14}$$

In the limit of small dissipation discussed previously, the unperturbed resonance frequencies of the flexural modes are given by $f_i = \frac{1}{2\pi}\sqrt{\frac{k_i}{m}}$. Similarly, the modified resonance frequencies are given by $f_i' = \frac{1}{2\pi}\sqrt{\frac{k_i'}{m}}$. For small tip-sample force derivatives, we apply (S.11) and (S.13) and arrive at $f_i' \approx \frac{1}{2\pi}\sqrt{\frac{k_i - F_{ii}}{m}}$. Expanding to first order in F_{ii}, we find $f_i' \approx f_i - \frac{f_i}{2k_i}F_{ii}$. Solving in terms of the frequency shift induced by the tip-sample interaction, we have:

$$\Delta f_i = f_i' - f_i \approx -\frac{f_i}{2k_i}F_{ii}. \tag{S.15}$$

We can now write a relation for $F_{ii} = \partial F_i/\partial r_i$ in terms of the measured frequency shift induced by the tip-sample interaction and properties of the unperturbed NW modes:

$$\frac{\partial F_i}{\partial r_i} \approx -2k_i\left(\frac{\Delta f_i}{f_i}\right). \tag{S.16}$$

Using (S.12) and (S.14), we can also write an expression involving the angle ϕ_i between the bare mode direction $\hat{\mathbf{r}}_i$ and the corresponding modified mode direction $\hat{\mathbf{r}}_i'$:

$$\tan\phi_i \approx \frac{F_{ij}}{|k_i - k_j|}. \tag{S.17}$$

This equation then allows us to solve for $F_{ij} = \partial F_i/\partial r_j$ for $i \neq j$ in terms of the measured angles ϕ_i and the unperturbed spring constants:

$$\frac{\partial F_i}{\partial F_j} \approx |k_i - k_j| \tan\phi_i. \tag{S.18}$$

In this way, we are able to measure all spatial tip-sample force derivatives in the small interaction limit (i.e. all derivatives much smaller than the unperturbed spring constants).

---

[1] Gloppe, A. *et al.* Bidimensional nano-optomechanics and topological backaction in a non-conservative radiation force field. *Nature Nanotech.* **9**, 920–926 (2014).

[2] Faust, T. *et al.* Nonadiabatic Dynamics of Two Strongly Coupled Nanomechanical Resonator Modes. *Phys. Rev. Lett.* **109,** 037205 (2012).