Cooling Mechanical Resonators

$$\hbar \omega >> k_B T$$

- Achieve ultimate force resolution
- Approach the quantum regime
- Measure mechanical superpositions and coherences

Superposition & Coherence?

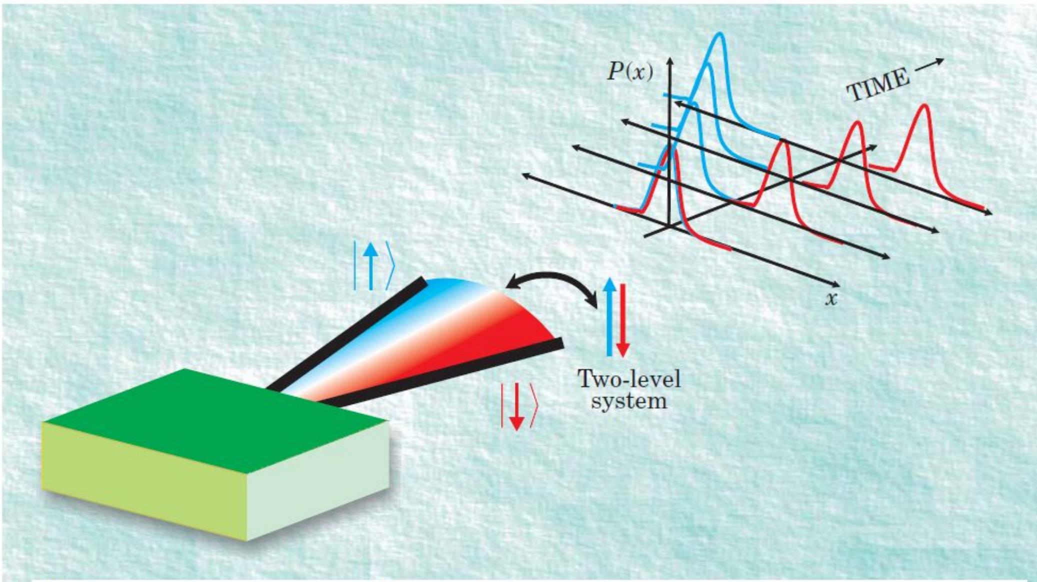


Figure 4. Quantum superpositions may be possible with a quantum two-level system (TLS) coupled to a nanomechanical resonator. In this schematic, the flexure of the cantilever depends on whether the TLS is in the down state |↓⟩ (red) or the up state |↑⟩ (blue). The plot shows the probability *P* of finding the resonator center-of-mass at a given position *x* for different times after coupling the resonator to the TLS. If the TLS is prepared in a superposition state, an entangled state is formed and the resonator will be in both locations.

Strategies for Cooling Resonators

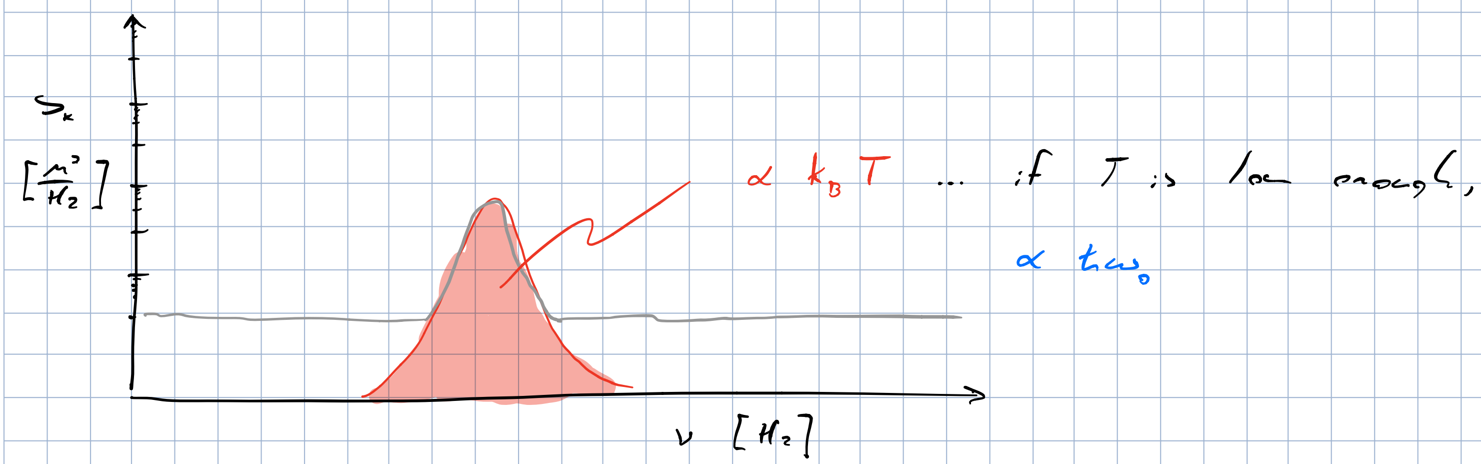
$$\hbar \omega \gg k_B T$$

- "Brute force": High resonance frequencies & low reservoir temperatures
- Damping mechanical motion
- Cavity cooling

Lecture 7 (28.10.2020)

I. Cooling Mechanical Resonators

In the next few lectures, we will discuss the cooling of mechanical resonators into their quantum mechanical ground state. What do we mean?



Let's first think of our mechanical mode of interest in our resonator as a harmonic oscillator, as usual. We can then calculate its zero-point energy using principles of statistical mechanics and quantum mechanics:

II. Zero-point Energy: Quantum Harmonic Osc.

$$E_n = \hbar\omega_0 \left(n + \frac{1}{2}\right) \quad \text{for} \quad n = 0, 1, 2, \ldots$$

$$E_0 = \frac{\hbar\omega_0}{2}$$ ← Zero-point energy

$$\langle x^2 \rangle = \langle n | \hat{x}^2 | n \rangle = \frac{\hbar}{m\omega_0}\left(n + \frac{1}{2}\right)$$

For $n=0$, i.e. in the ground state:

$$\langle x^2 \rangle_0 = \frac{\hbar}{2m\omega_0} = x_{ZPF}^2$$ ← Zero-point fluctuations

Standard Quantum Limit:

$$\Delta x_{SQL} = x_{ZPF} = \sqrt{\langle x^2 \rangle_0} = \sqrt{\frac{\hbar}{2m\omega_0}}$$

Wave function for $n=0$ of harmonic oscillator:

$$\psi_0(x) = \left(\frac{1}{2\pi\, x_{ZPF}^2}\right)^{1/4} e^{-\left(\frac{x}{2\, x_{ZPF}}\right)^2}$$

According to statistical mechanics what is the mean energy $\bar{E}$ in a quantum harmonic oscillator at temperature T?

$$\bar{E} = \frac{\sum_{n=0}^{\infty} e^{-\beta E_n} \cdot E_n}{\sum_{n=0}^{\infty} e^{-\beta E_n}}, \quad \text{where } \beta = \frac{1}{k_B T}$$

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{\partial}{\partial \beta}\ln Z,$$

where $Z = \sum_{n=0}^{\infty} e^{-\beta E_n}$

$$Z = e^{-\frac{1}{2}\beta\hbar\omega_0} \sum_{n=0}^{\infty} e^{-n\beta\hbar\omega_0} = e^{-\frac{1}{2}\beta\hbar\omega_0}\left(1 + e^{-\beta\hbar\omega_0} + \dots\right)$$

$$Z = e^{-\frac{1}{2}\beta\hbar\omega_0}\left(\frac{1}{1 - e^{-\beta\hbar\omega_0}}\right)$$

$$\ln Z = -\frac{1}{2}\beta\hbar\omega_0 - \ln\left(1 - e^{-\beta\hbar\omega_0}\right)$$

$\therefore$

$$\bar{E} = -\frac{\partial}{\partial\beta}\ln Z = -\left(-\frac{1}{2}\hbar\omega_0 - \frac{e^{-\beta\hbar\omega_0}\,\hbar\omega_0}{1 - e^{-\beta\hbar\omega_0}}\right)$$

$$\boxed{\bar{E} = \hbar\omega_0\left(\frac{1}{2} + \frac{1}{e^{\frac{\hbar\omega_0}{k_B T}} - 1}\right)}$$

For $k_B T >> \hbar\omega_0$:

$$\bar{E} \approx \hbar\omega_0\left(\frac{1}{2} + \frac{k_B T}{\hbar\omega_0}\right)$$

$$\boxed{\bar{E} \approx k_B T}$$ ← Equipartition

For $k_B T << \hbar\omega_0$:

$$\boxed{\bar{E} \approx \hbar\omega_0\left(\frac{1}{2} + e^{-\frac{\hbar\omega_0}{k_B T}}\right)}$$ ← $\lim_{T\to 0} \bar{E} = E_0$

III. Zero-point motion : Quantum Harmonic Osc.

$$\langle x^2 \rangle = \langle n | \hat{x}^2 | n \rangle = \frac{\hbar}{m\omega_0}\left(n + \frac{1}{2}\right) = \frac{E_n}{m\omega_0^2}$$

$$\overline{\langle x^2 \rangle} = \frac{\bar{E}}{m\omega_0^2}$$

$$\langle \overline{x^2} \rangle = \frac{\hbar}{m\omega_0}\left(\frac{1}{2} + \frac{1}{e^{\frac{\hbar\omega_0}{k_B T}} - 1}\right)$$

$$\langle \overline{x^2} \rangle = x_{ZPF}^2 + 2x_{ZPF}^2\left(\frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1}\right)$$

So how do we put nanomechanical oscillators into a quantum state, specifically the ground state. One way is by brute force : picking a high-frequency mode and putting it in a cold fridge, such that :

$$\hbar\omega_0 \gg k_B T \quad .$$

Alternatively, one can try to cool the mechanical mode of interest – rather than the entire bath – by a number of techniques. One of these is feedback cooling :

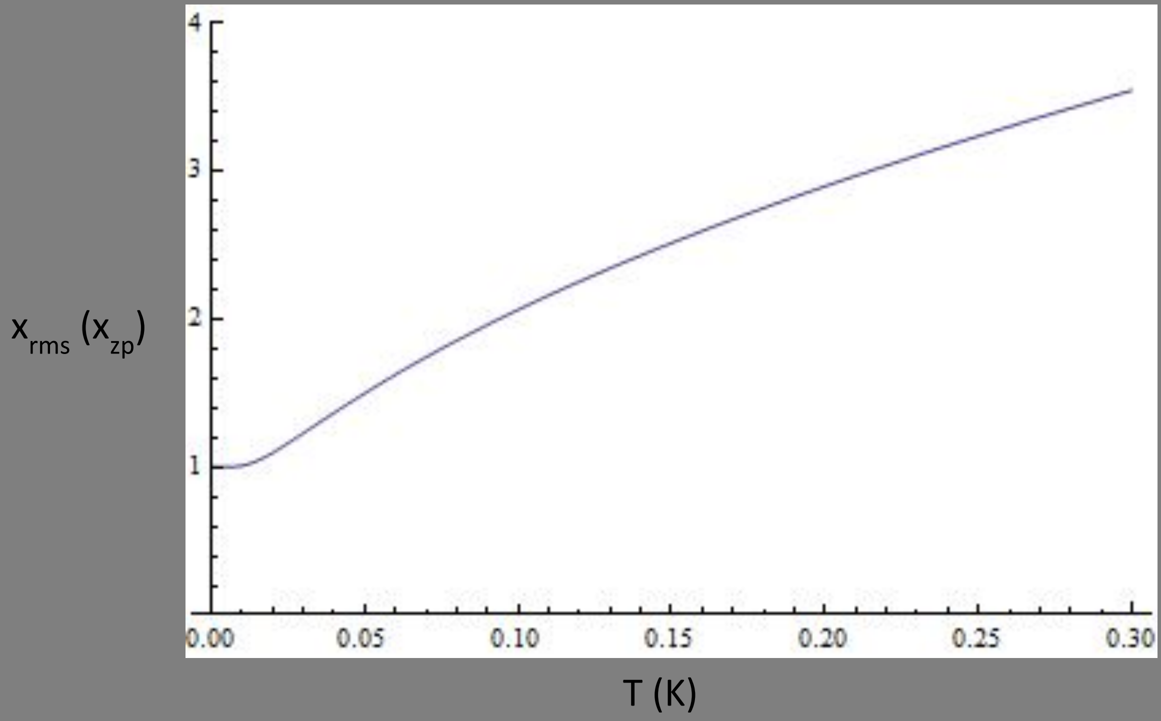
$$x_{rms} = \sqrt{x_{zp}^2 + 2x_{zp}^2 \left(\frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1} \right)}$$

$\hbar\omega >> k_B T$

$$x_{rms} = x_{zp} = \sqrt{\frac{\hbar}{2m\omega}}$$

$\hbar\omega << k_B T$

$$x_{rms} = x_{th} = \sqrt{\frac{k_B T}{m\omega^2}}$$



“Brute Force”

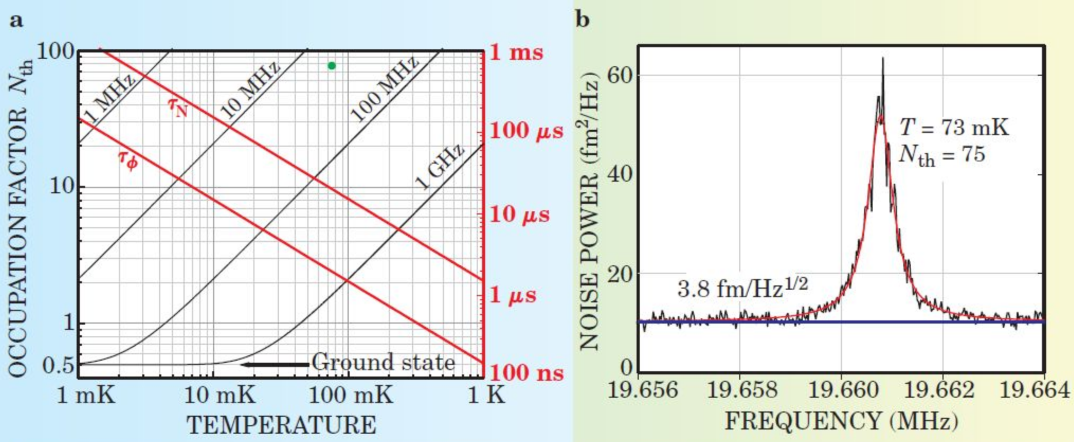


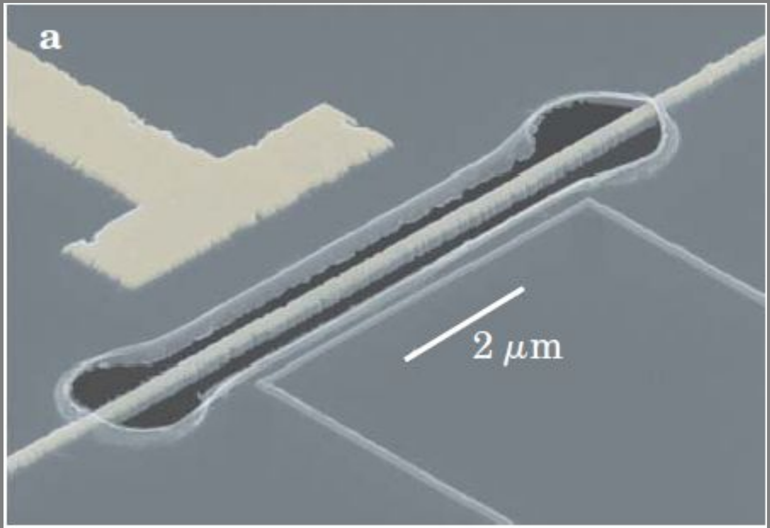
Figure 2. Quantum limits. (a) The occupation factor N_{th} (black curves) of various mechanical resonator frequencies is a function of resonant frequency and temperature T. Shown in red is the lifetime τ_N of a given number state for a 10-MHz resonator with quality factor Q = 200 000 (recently demonstrated at the Laboratory for Physical Sciences).[11] Also in red is the expected decoherence time τ_ϕ for a superposition of two coherent states in that resonator displaced by 100 fm. **(b)** The measured noise-power spectrum of the thermal motion (black line, with a Lorentzian fit in red) atop the white noise (blue baseline) of the position detector. The curve corresponds to the green point in panel a, with T = 73 mK and N_{th} = 75. These data show the closest approach to date to the uncertainty-principle limit: The detector noise gives a displacement sensitivity a factor of 5.8 from the quantum mechanical limit.

Real Numbers (T = 1 K)

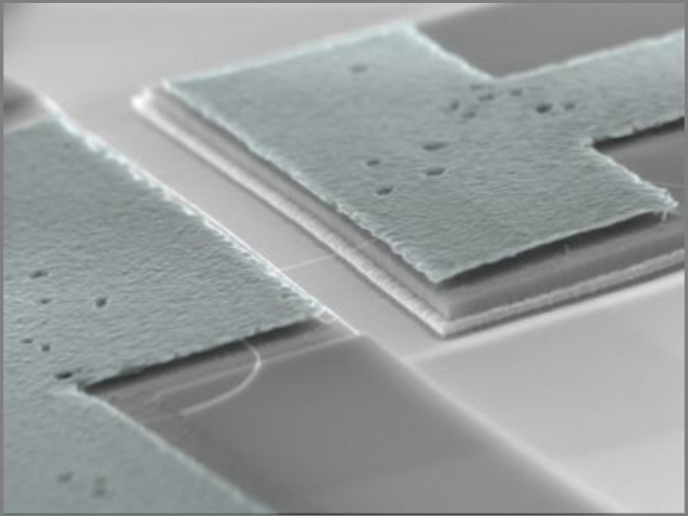
Top-down doubly clamped beams (Schwab)

- $m = 10^{-15}$ kg
- $\omega = 2\pi \times 10$ MHz

- $x_{th} = 2 \times 10^{-12}$ m
- $x_{zp} = 3 \times 10^{-14}$ m



Real Numbers (T = 1 K)



Bottom-up doubly clamped "clean" nanotubes (Steele/Delft)

- $m = 10^{-21}$ kg
- $\omega = 2\pi \times 500$ MHz

- $x_{th} = 4 \times 10^{-11}$ m
- $x_{zp} = 4 \times 10^{-12}$ m

Real Numbers (T = 1 K)

Top-down doubly clamped beams (Schwab)

- $m = 10^{-15}$ kg
- $\omega = 2\pi \times 10$ MHz

- $x_{th} = 2 \times 10^{-12}$ m
- $x_{zp} = 3 \times 10^{-14}$ m

Bottom-up doubly clamped "clean" nanotubes (Steele/Delft)

- $m = 10^{-21}$ kg
- $\omega = 2\pi \times 500$ MHz

- $x_{th} = 4 \times 10^{-11}$ m
- $x_{zp} = 4 \times 10^{-12}$ m

Real Numbers (T = 10 mK)

Top-down doubly clamped Si beams (Schwab)

- $m = 10^{-15}$ kg
- $\omega = 2\pi \times 10$ MHz

- $x_{th} = 2 \times 10^{-13}$ m
- $x_{zp} = 3 \times 10^{-14}$ m

Bottom-up doubly clamped "clean" nanotubes (Steele/Delft)

- $m = 10^{-21}$ kg
- $\omega = 2\pi \times 500$ MHz

- $x_{th} = 4 \times 10^{-12}$ m
- $x_{zp} = 4 \times 10^{-12}$ m

Technical Challenges

- Resonator Fabrication (high frequency, low dissipation, low mass)
- Displacement sensing (low measurement imprecision, i.e. low noise floor)
- Refrigeration (mK temperatures)

Approaching the Quantum Limit of a Nanomechanical Resonator

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By coupling a single-electron transistor to a high–quality factor, 19.7-megahertz nanomechanical resonator, we demonstrate position detection approaching that set by the Heisenberg uncertainty principle limit. At millikelvin temperatures, position resolution a factor of 4.3 above the quantum limit is achieved and demonstrates the near-ideal performance of the single-electron transistor as a linear amplifier. We have observed the resonator's thermal motion at temperatures as low as 56 millikelvin, with quantum occupation factors of N_{TH} = 58. The implications of this experiment reach from the ultimate limits of force microscopy to qubit readout for quantum information devices.

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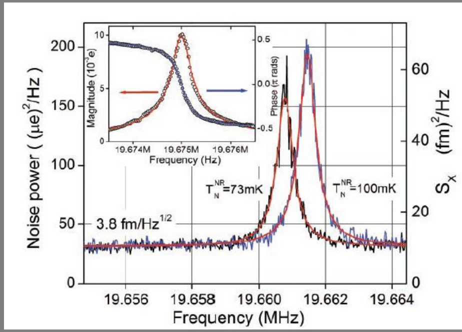
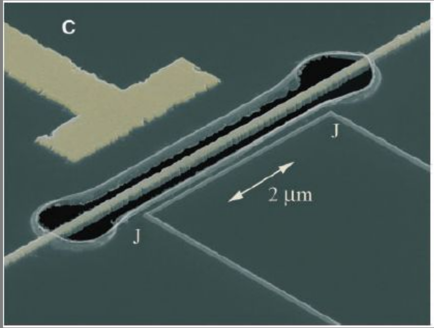
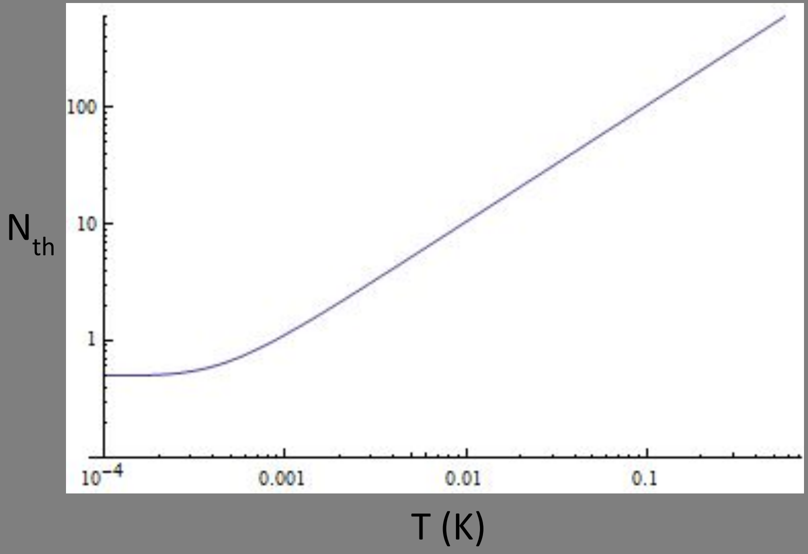
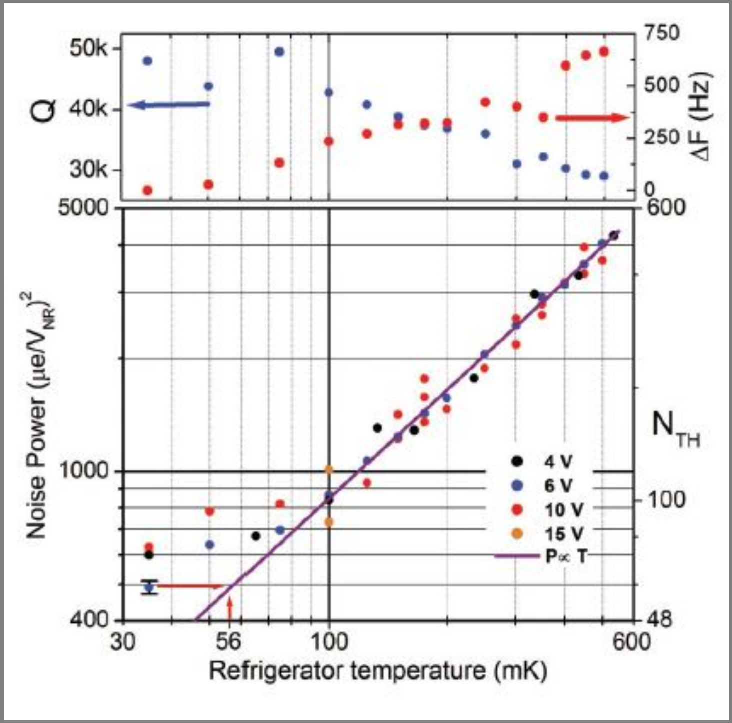




Expectation vs. Reality





Strategies for Cooling Resonators

$$\hbar \omega >> k_B T$$

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