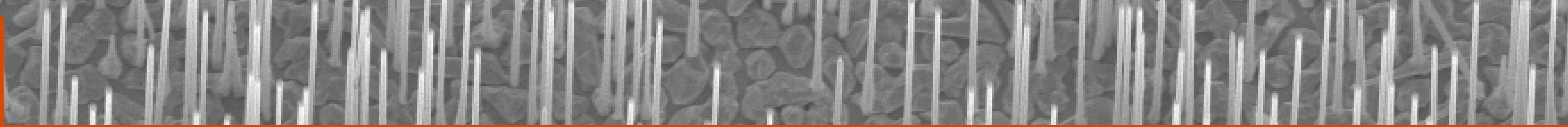


# Mechanics of Nanowires

Introduction to Nanomechanics - Fall 2021


$$S_F = 4\,k_B T \Gamma$$

$$\Gamma = \frac{m\omega_0}{Q} \qquad \frac{dE}{dt} = -\Gamma \dot{x}^2$$

$$m \propto wtl \qquad \omega_0 \propto \frac{t}{l^2} \qquad Q \propto t$$

$$\Gamma \propto \frac{wt}{l}$$

Scale all lengths by $\beta$:

$$\Gamma \propto \beta \qquad \omega_0 \propto \beta^{-1}$$

Amplitude measurement:

$$F_{min} = \sqrt{4\ k_B T \Gamma} \propto \sqrt{\frac{wt}{l}} \propto \beta^{1/2}$$

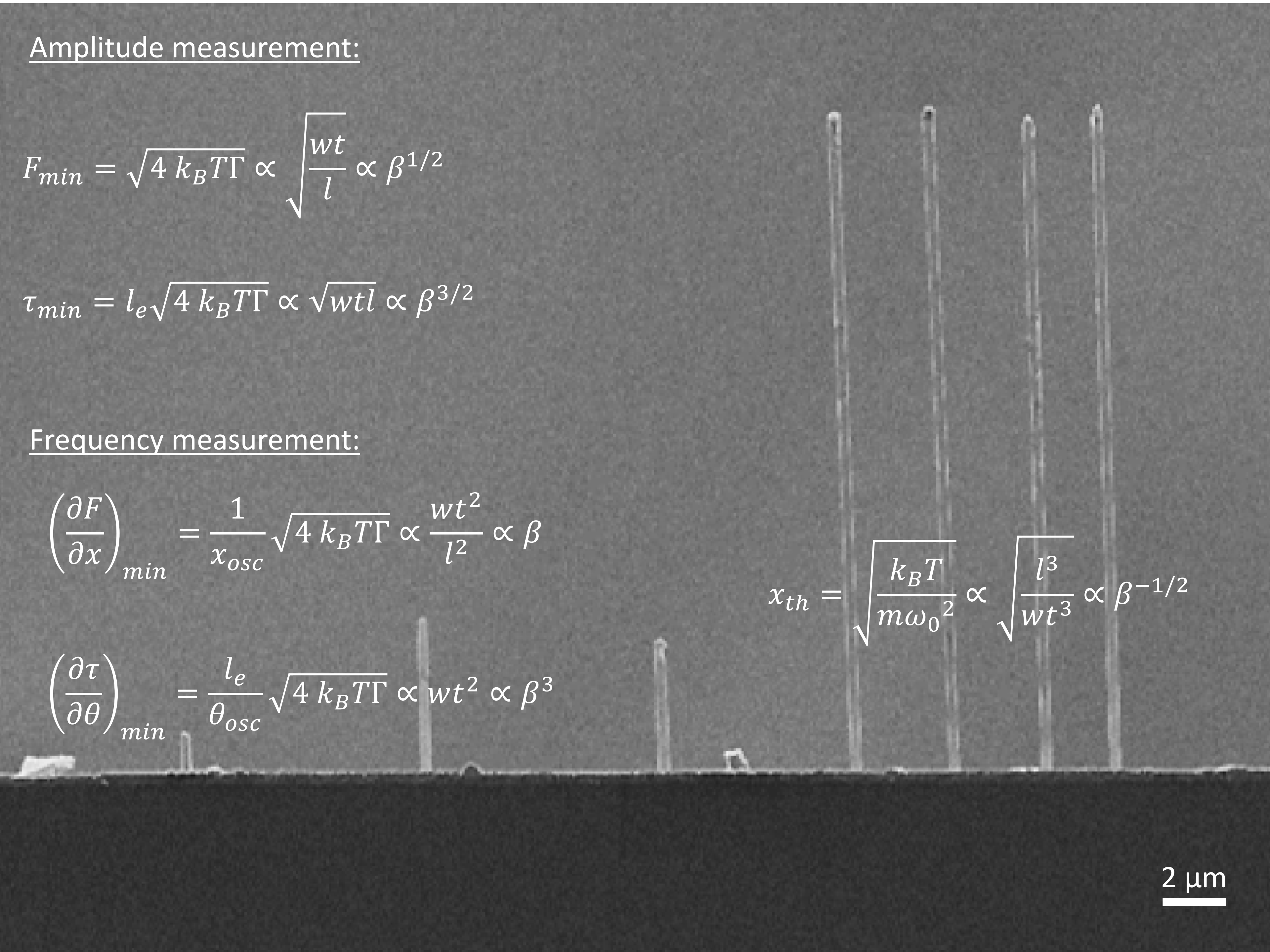
$$\tau_{min} = l_e \sqrt{4\ k_B T \Gamma} \propto \sqrt{wtl} \propto \beta^{3/2}$$

Frequency measurement:

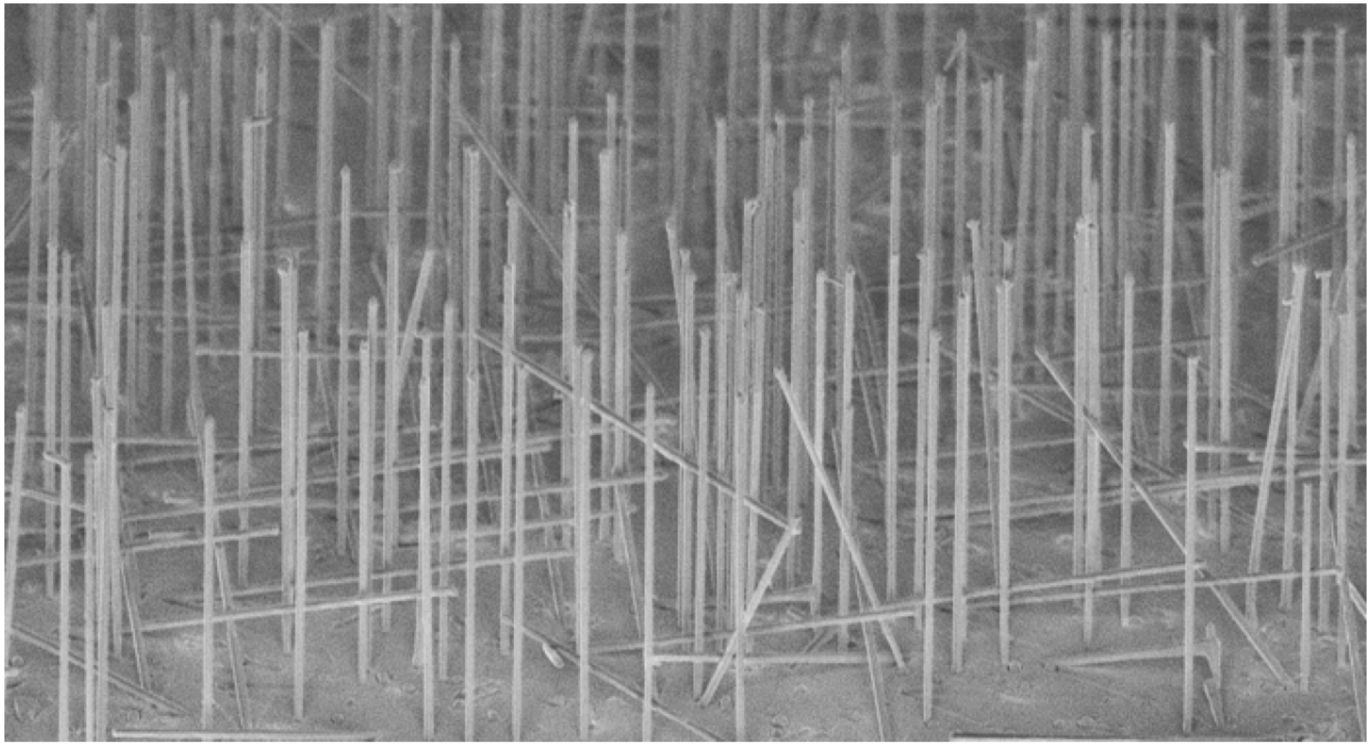
$$\left(\frac{\partial F}{\partial x}\right)_{min} = \frac{1}{x_{osc}} \sqrt{4\ k_B T \Gamma} \propto \frac{wt^2}{l^2} \propto \beta$$

$$\left(\frac{\partial \tau}{\partial \theta}\right)_{min} = \frac{l_e}{\theta_{osc}} \sqrt{4\ k_B T \Gamma} \propto wt^2 \propto \beta^3$$

$$x_{th} = \sqrt{\frac{k_B T}{m{\omega_0}^2}} \propto \sqrt{\frac{l^3}{wt^3}} \propto \beta^{-1/2}$$

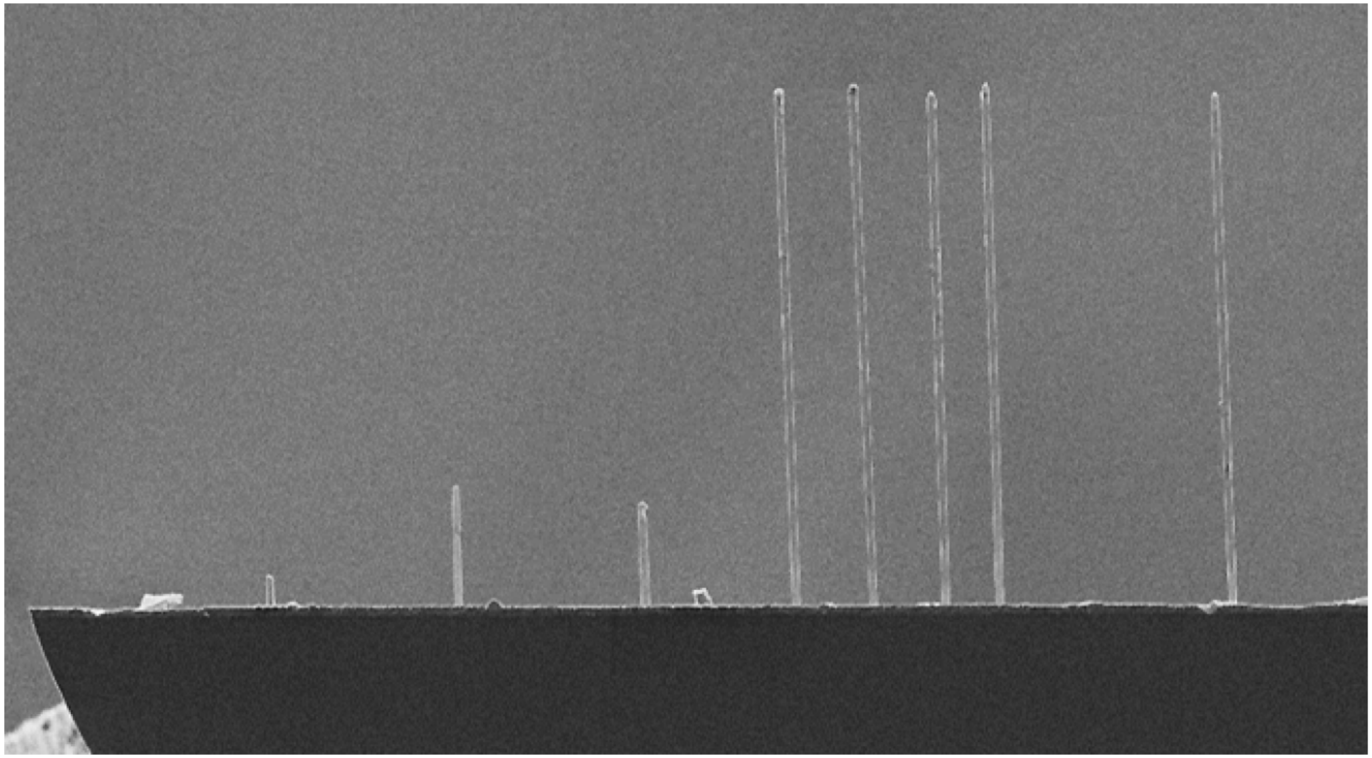


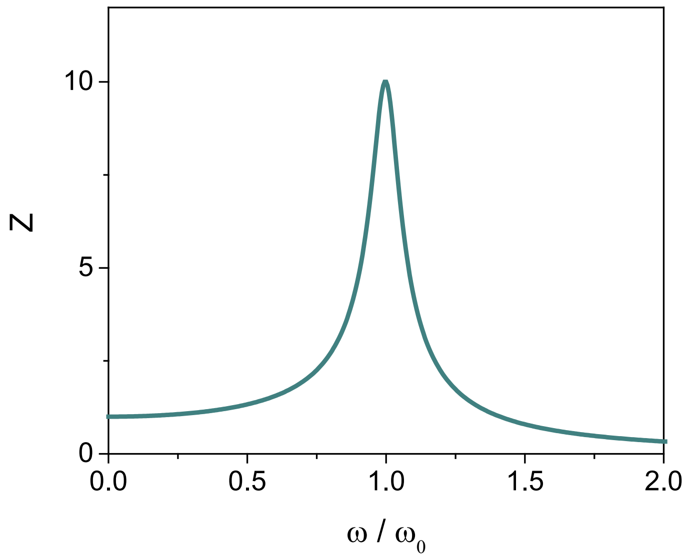
# MBE-grown GaAs/AlGaAs nanowires



ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

# MBE-grown GaAs/AlGaAs nanowires






**Equation of motion**

$$\ddot{x}(t) + \gamma \dot{x}(t) + \omega_0 x(t) = F(t)$$

# Duffing Oscillator

**Duffing Equation**

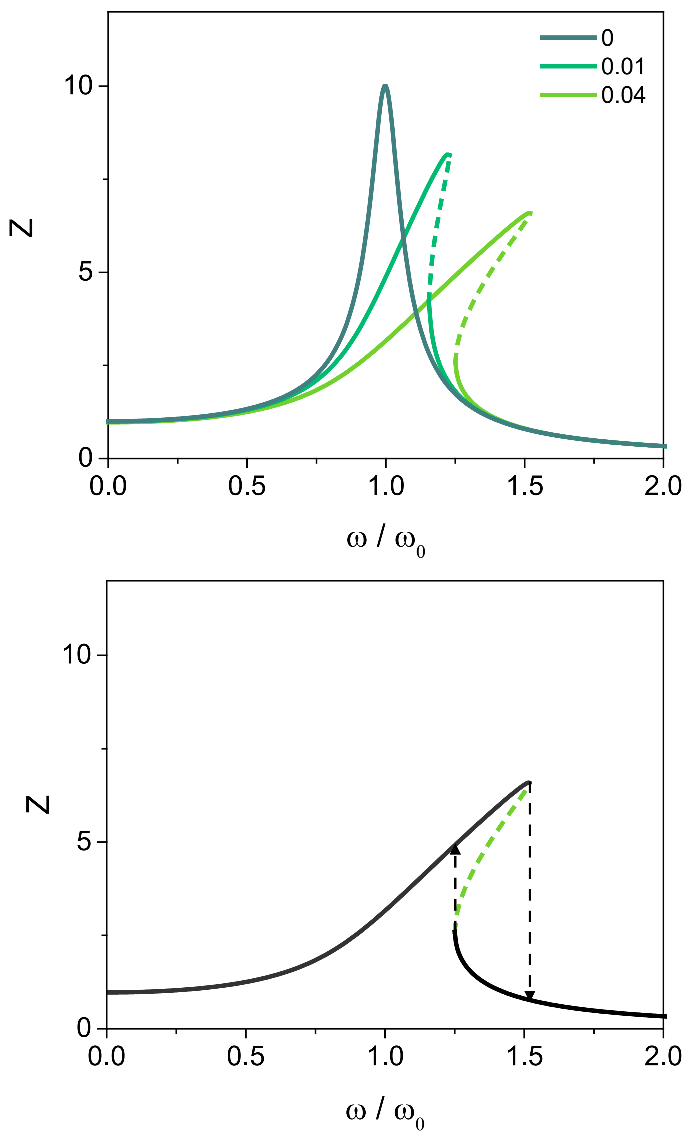
$$\ddot{x}(t) + \gamma\dot{x}(t) + \omega_0 x(t) + \alpha x^3(t) = F(t)$$

**positive** (negative) α could be seen as a **hardening** (softening) of the spring constant

$$\ddot{x}(t) + \gamma\dot{x}(t) + x(t)(\omega_0 + \alpha x^2(t)) = F(t)$$

$$x(t) = Z\cos(\omega t - \psi)$$

# Duffing Oscillator



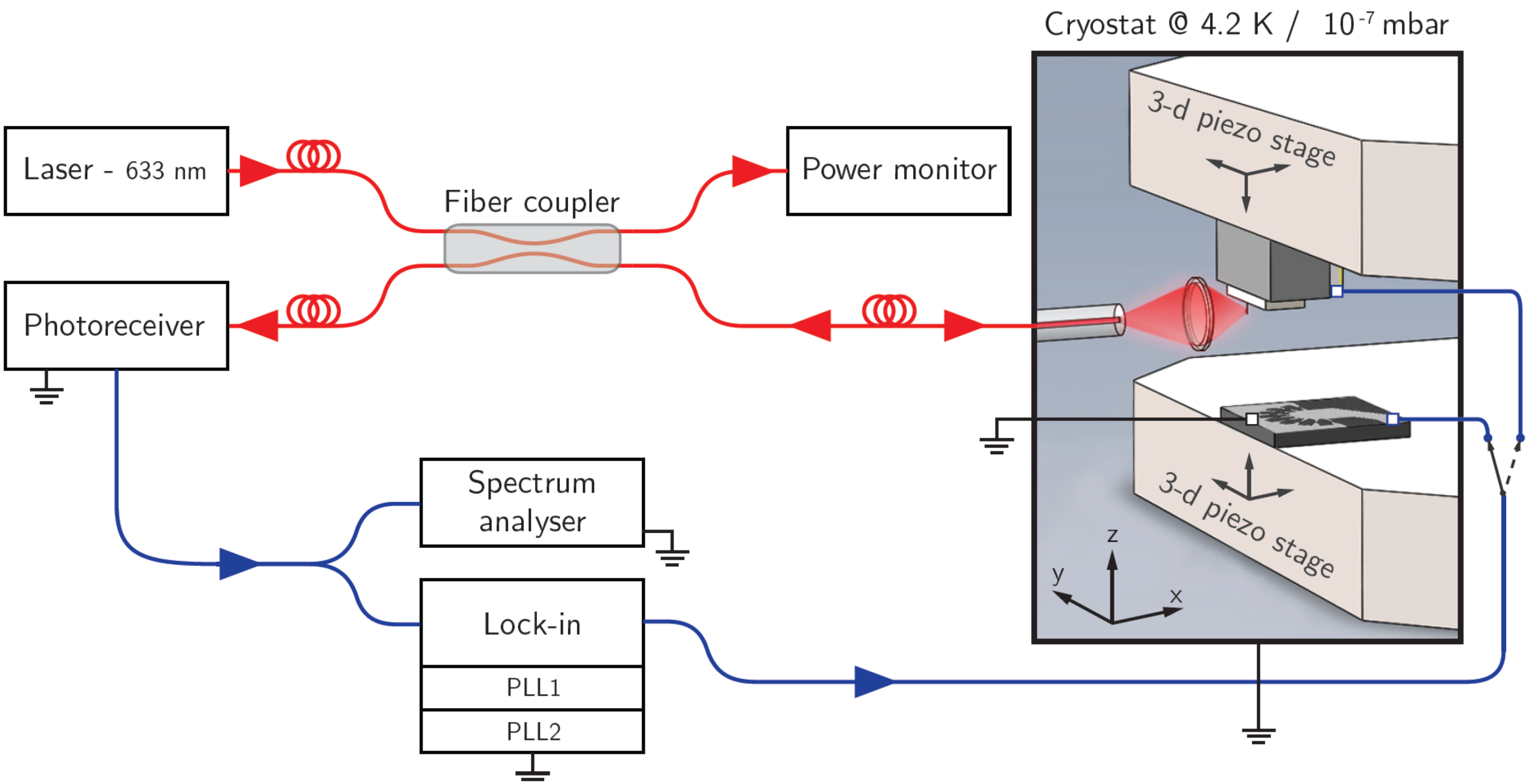

**Duffing Equation**

$$Z^2\left(\omega^2-\omega_0^2-\frac{3}{4}\alpha Z^2\right)^2+(\gamma Z\,\omega)^2=\hat{F}$$

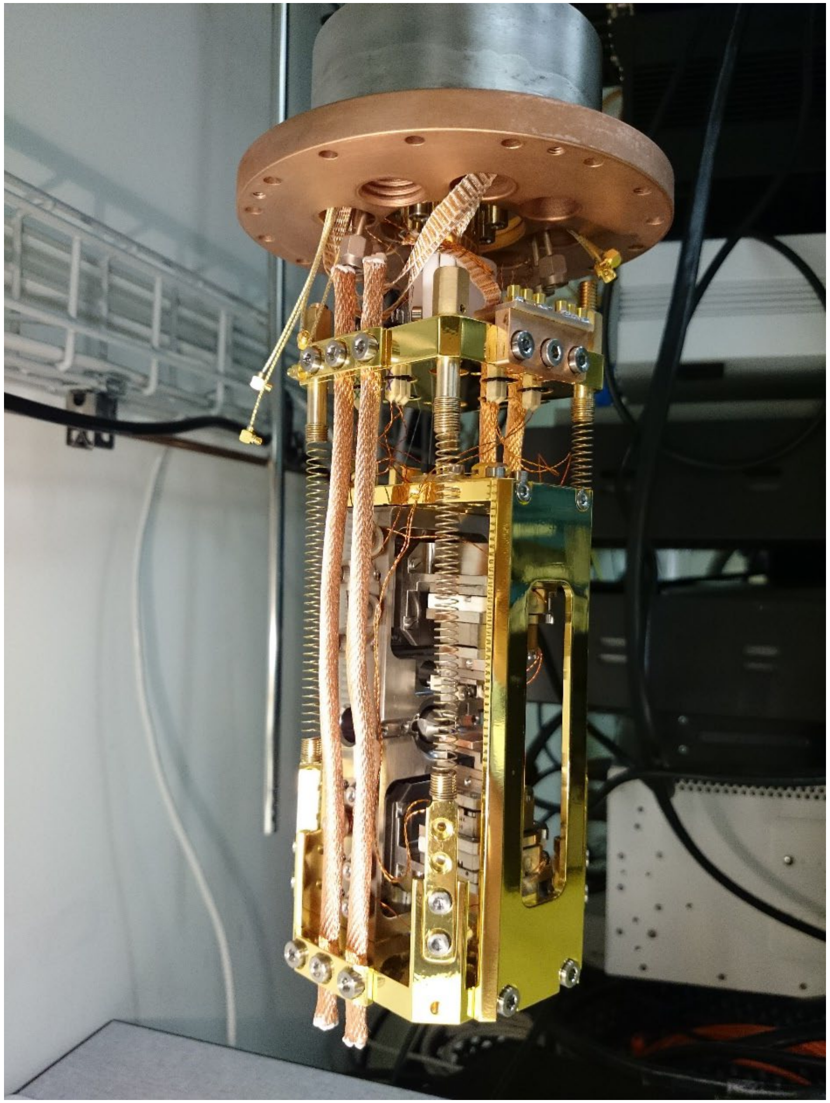
**Bistable solution!**

The jumping point depends on the history of the resonator

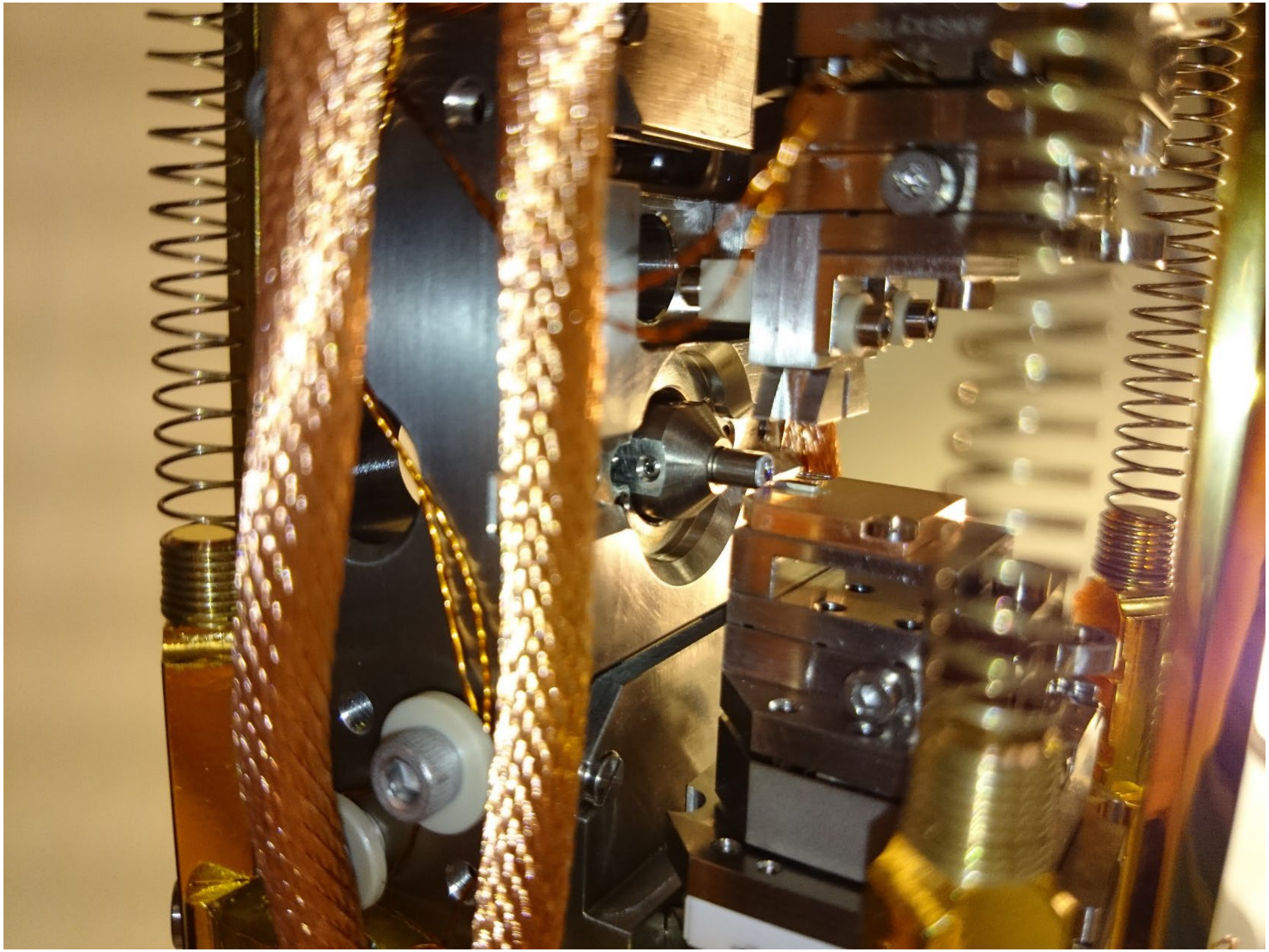
# Low-temperature scanning NW setup



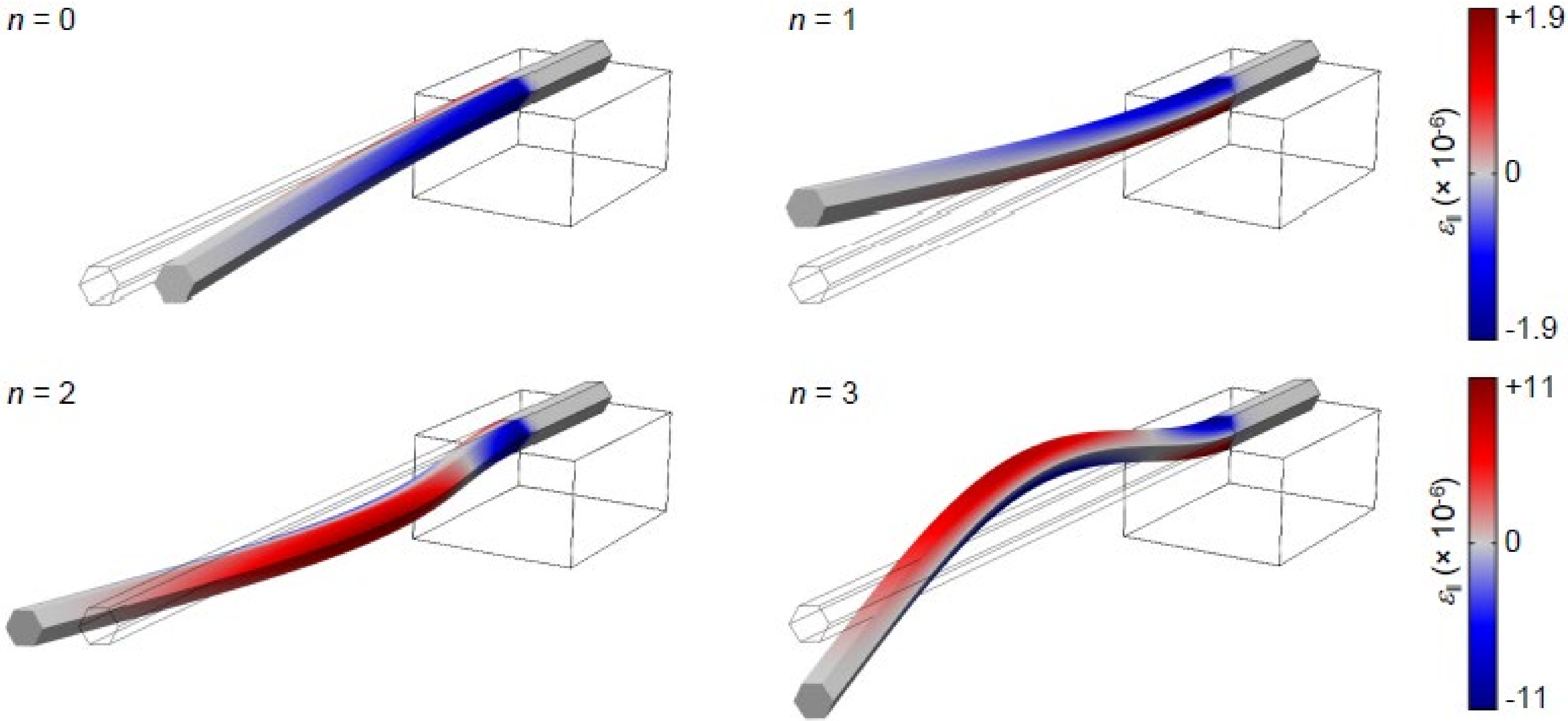
# Low-temperature scanning NW setup



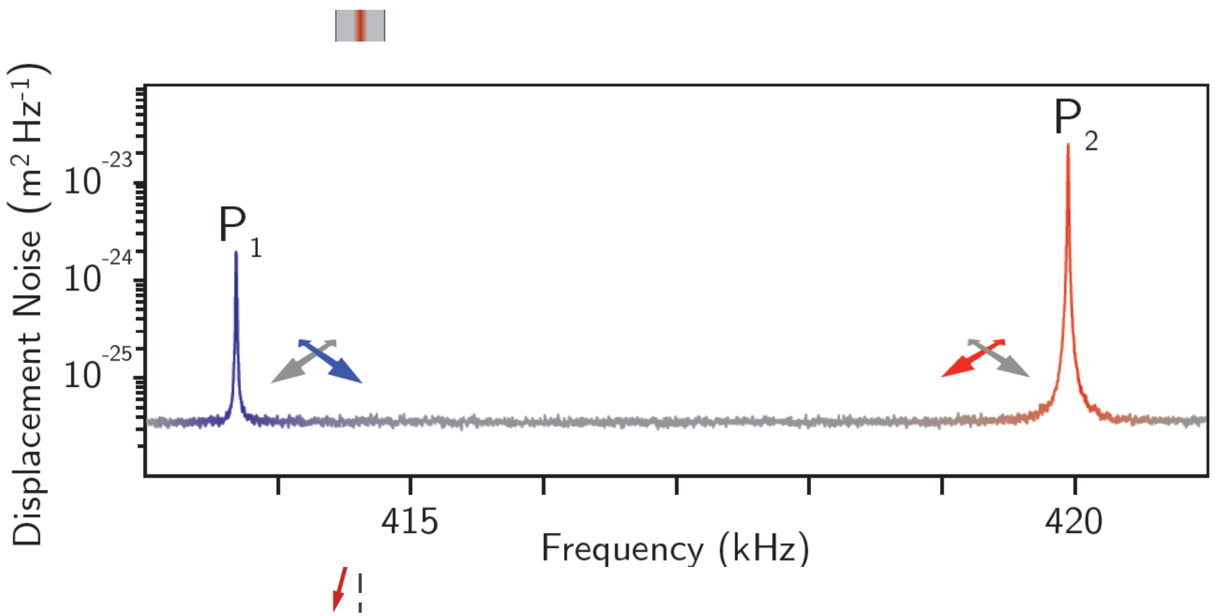
# Low-temperature scanning NW setup



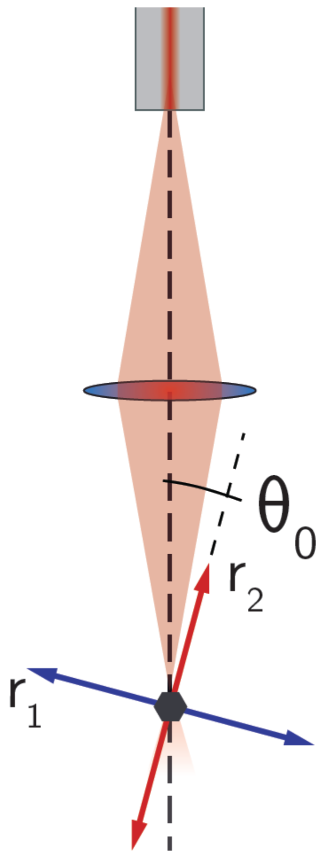
# Mechanical modes



# Nanowire mode doublet



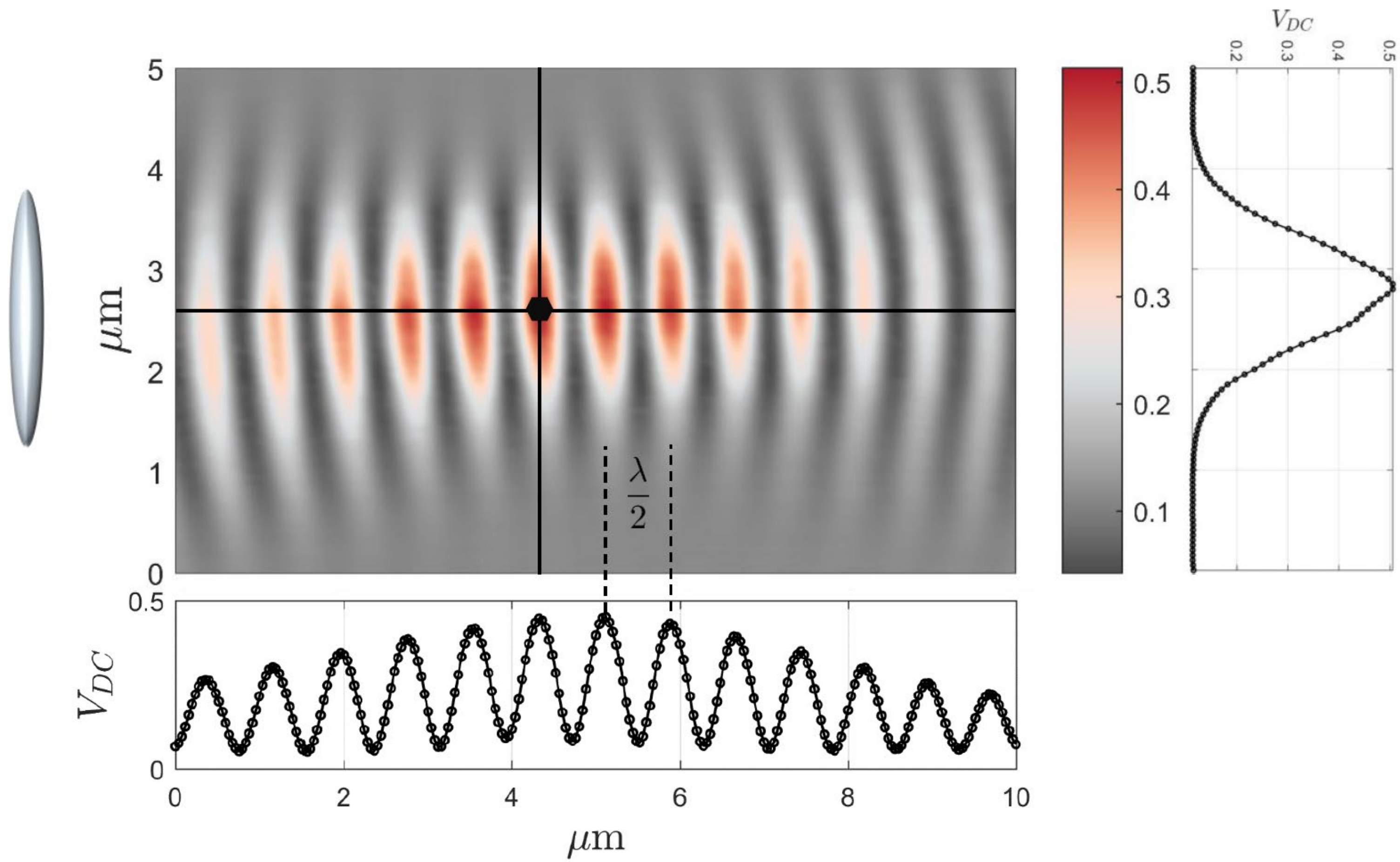
# Interferometric detection




**Oscillation direction estimation**

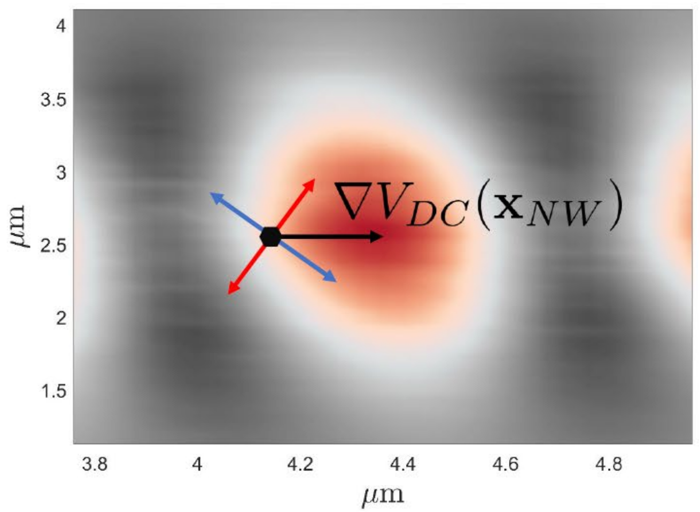
- Mean square displacement: $\langle x^2 \rangle = P_1 + P_2 = \langle r_1^2 \rangle \sin^2\theta_0 + \langle r_2^2 \rangle \cos^2\theta_0$
- Angle: $\theta_0 = \arctan(\frac{f_1}{f_2}\sqrt{\frac{P_1}{P_2}})$

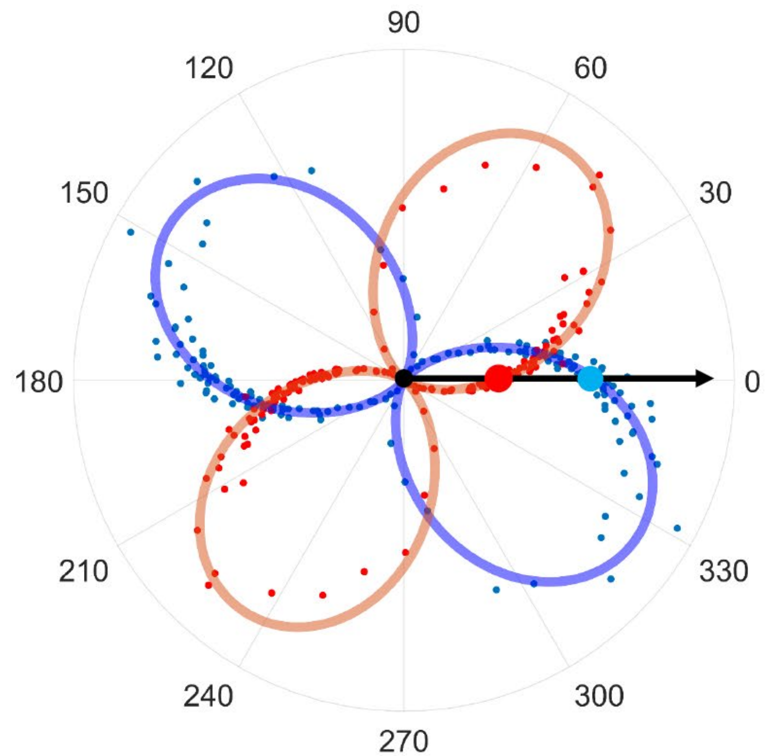
# Interferometric measurement of NW motion




A. Gloppe et al. In: *Nat Nano* 9 (2014), pp. 920–926.

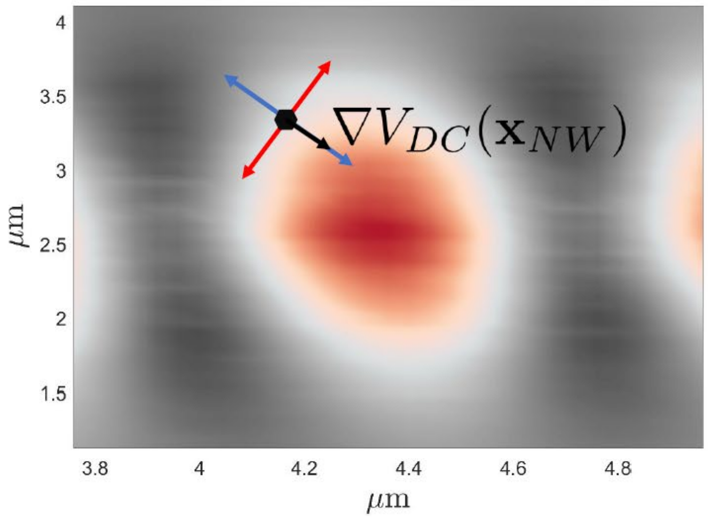
# Interferometric measurement of NW motion

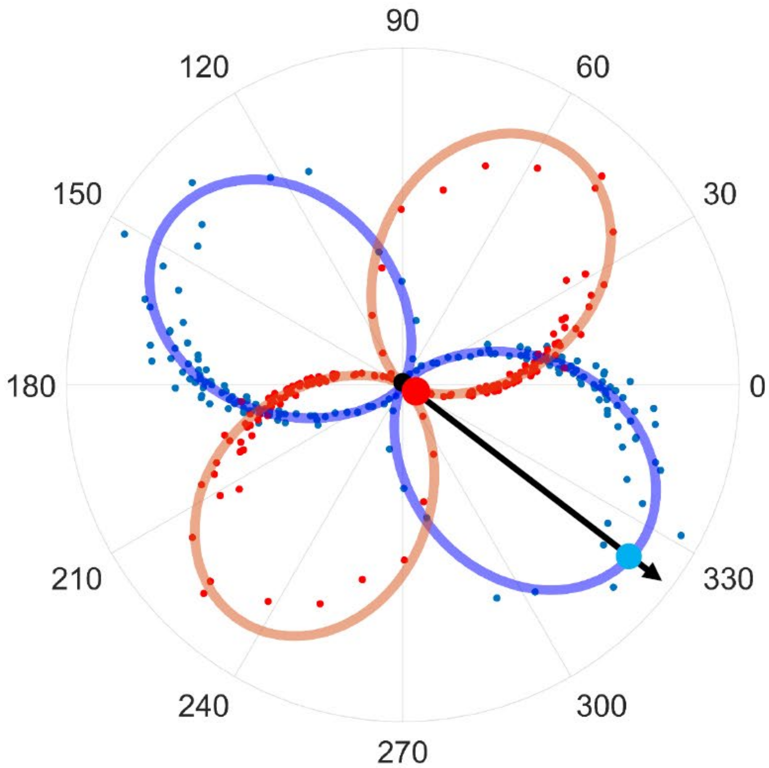



A. Gloppe et al. In: *Nat Nano* 9 (2014), pp. 920–926.

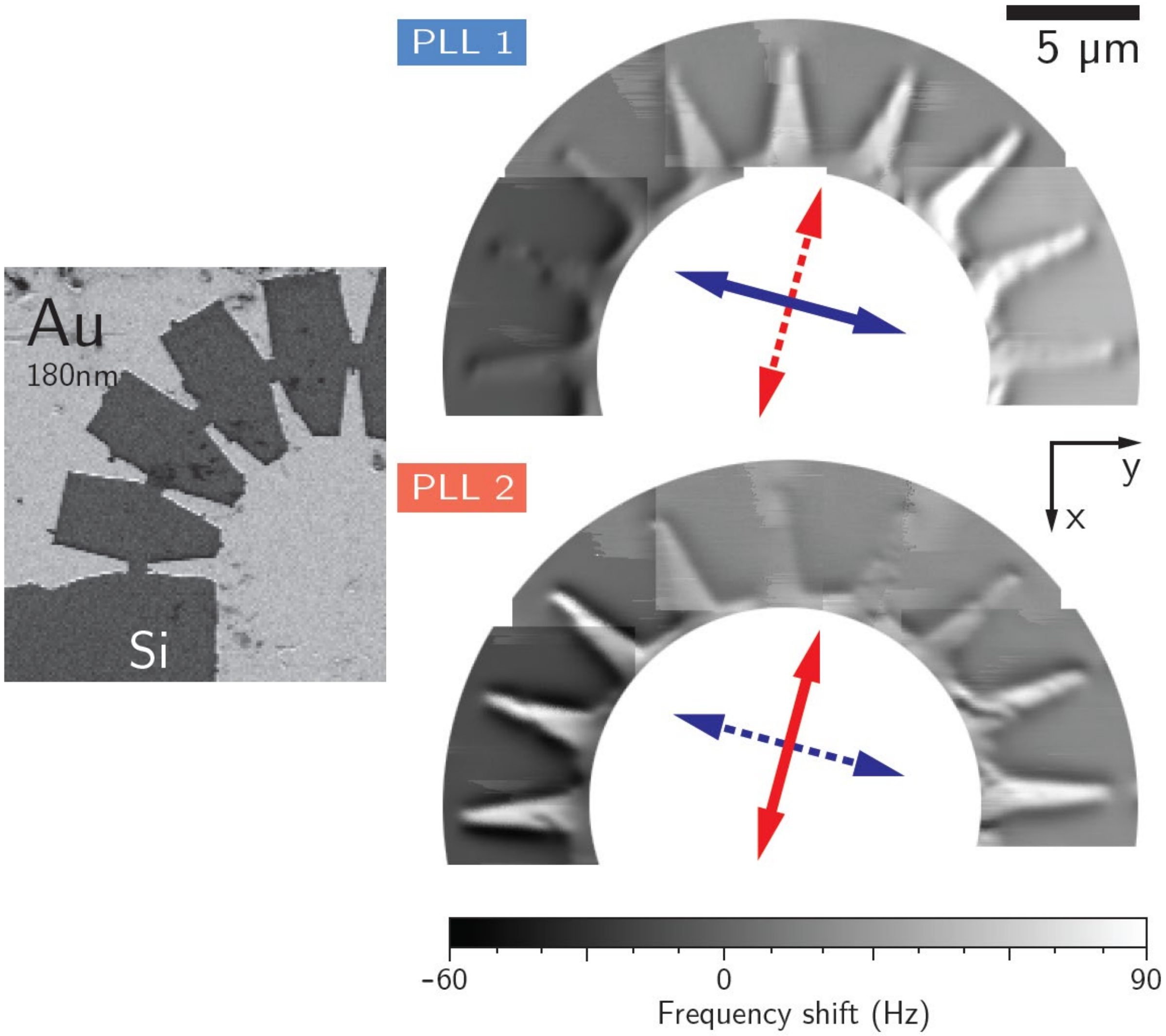
# Interferometric measurement of NW motion



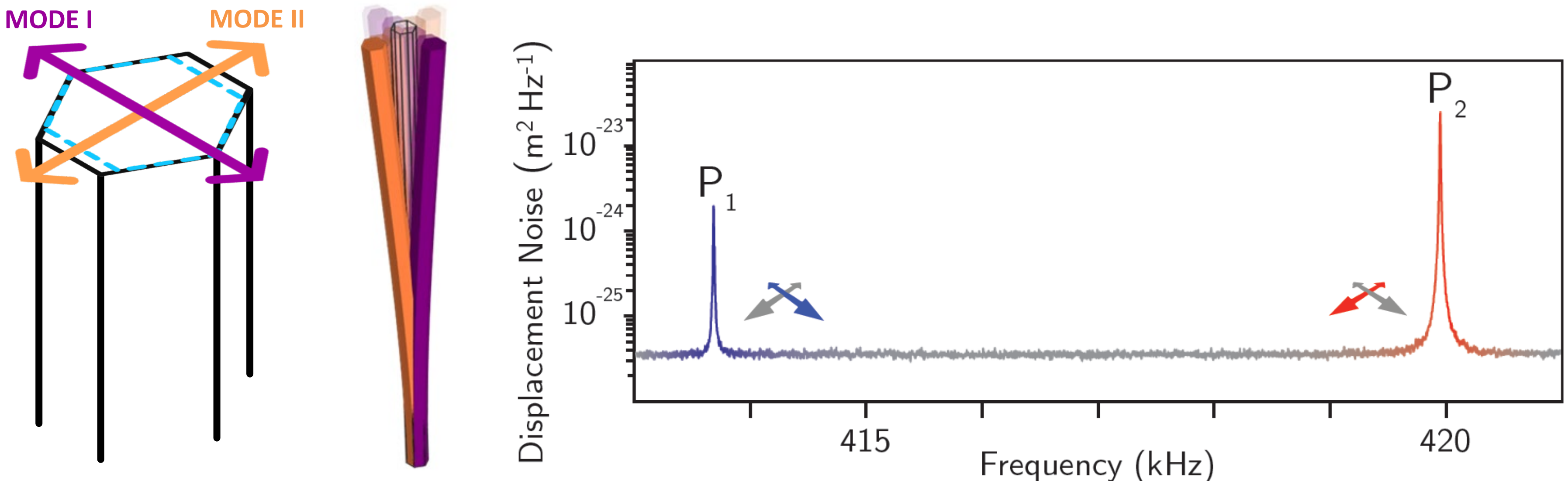

A. Gloppe et al. In: *Nat Nano* 9 (2014), pp. 920–926.

# Two-mode lateral force microscopy




O. Pfeiffer et al., *Phys. Rev. B* **65**, 161403R (2002)

# Nanowire mode doublet




- Slightly asymmetric nanowire gives two non-degenerate flexural modes

$$\omega_0 = \beta_n \sqrt{\frac{5EA}{24m}\frac{\mathrm{d}}{\mathrm{L}^2}}$$

# External Force

$$m\ddot{r}_i + \Gamma_i \dot{r}_i + k_i r_i = F_{th} + F_i$$

$$F_i \approx F_i(0) + r_j \frac{\partial F_i}{\partial r_j}\bigg|_0$$

$$m\ddot{r}_i + \Gamma_i \dot{r}_i + k_i r_i = F_{th} + F_i(0) + F_{ij} r_j$$

# External Force

$$m\ddot{\mathbf{r}} + \bar{\Gamma} \cdot \dot{\mathbf{r}} + \bar{K} \cdot \mathbf{r} = \mathbf{F}_{th} + \mathbf{F}_0;$$

$$\bar{\Gamma} \equiv \begin{pmatrix} \Gamma_1 & 0 \\ 0 & \Gamma_2 \end{pmatrix},$$

$$\bar{K} \equiv \begin{pmatrix} k_1 - F_{11} & -F_{21} \\ -F_{12} & k_2 - F_{22} \end{pmatrix}$$

# External Force

$$k_1' = \frac{1}{2}\left[k_1 + k_2 - F_{11} - F_{22} + \sqrt{(k_1 - k_2 - F_{11} + F_{22})^2 + 4F_{12}F_{21}}\right]$$

$$\hat{\mathbf{r}}_1' = \frac{1}{\sqrt{(k_2 - F_{22} - k_1')^2 + F_{12}^2}} \begin{pmatrix} k_2 - F_{22} - k_1' \\ F_{12} \end{pmatrix};$$

$$k_2' = \frac{1}{2}\left[k_1 + k_2 - F_{11} - F_{22} - \sqrt{(k_1 - k_2 - F_{11} + F_{22})^2 + 4F_{12}F_{21}}\right]$$

$$\hat{\mathbf{r}}_2' = \frac{1}{\sqrt{(k_1 - F_{11} - k_2')^2 + F_{21}^2}} \begin{pmatrix} F_{21} \\ k_1 - F_{11} - k_2' \end{pmatrix}.$$

# External Force

$$k_1' \approx k_1 - F_{11},$$

$$\hat{\mathbf{r}}_1' \approx \frac{1}{\sqrt{(k_1 - k_2)^2 + F_{12}^2}} \begin{pmatrix} k_1 - k_2 \\ -F_{12} \end{pmatrix}$$

$$k_2' \approx k_2 - F_{22},$$

$$\hat{\mathbf{r}}_2' \approx \frac{1}{\sqrt{(k_1 - k_2)^2 + F_{21}^2}} \begin{pmatrix} F_{21} \\ k_1 - k_2 \end{pmatrix}$$

# External Force

$$\Delta f_i = f_i' - f_i \approx -\frac{f_i}{2k_i} F_{ii} \qquad \frac{\partial F_i}{\partial r_i} \approx -2k_i \left( \frac{\Delta f_i}{f_i} \right)$$

$$\tan \phi_i \approx \frac{F_{ij}}{|k_i - k_j|} \qquad \frac{\partial F_i}{\partial F_j} \approx |k_i - k_j| \tan \phi_i$$

# In-plane spatial force derivatives

- NW tip equations of motion:

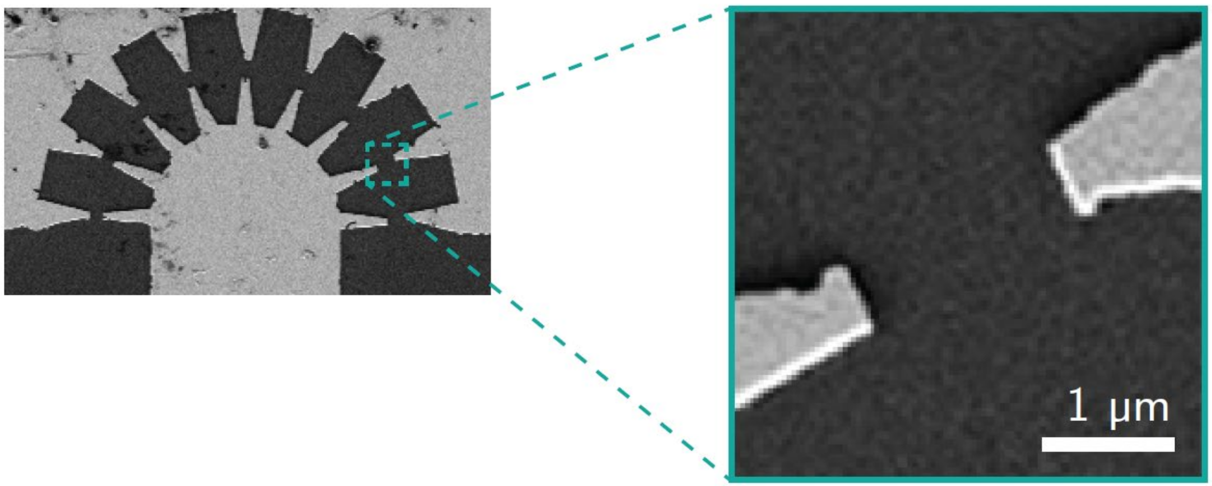
$$m\ddot{r}_i + \Gamma_i \dot{r}_i + k_i r_i = F_{th} + F_i$$

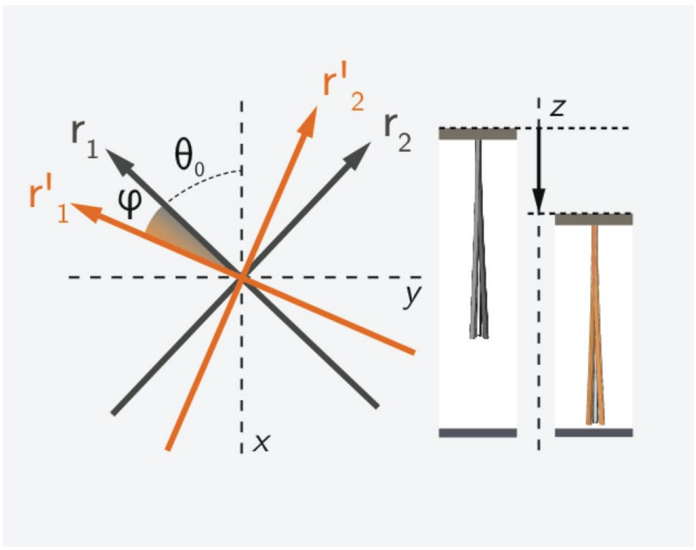
- For small oscillations and $\frac{\partial F}{\partial r} \ll k_{1,2}$:

$$\frac{\partial F_i}{\partial r_i} \approx -2k_i \frac{\Delta f_i}{f_i}$$

$$\frac{\partial F_i}{\partial r_j} \approx \left|k_i - k_j\right| \tan\phi$$

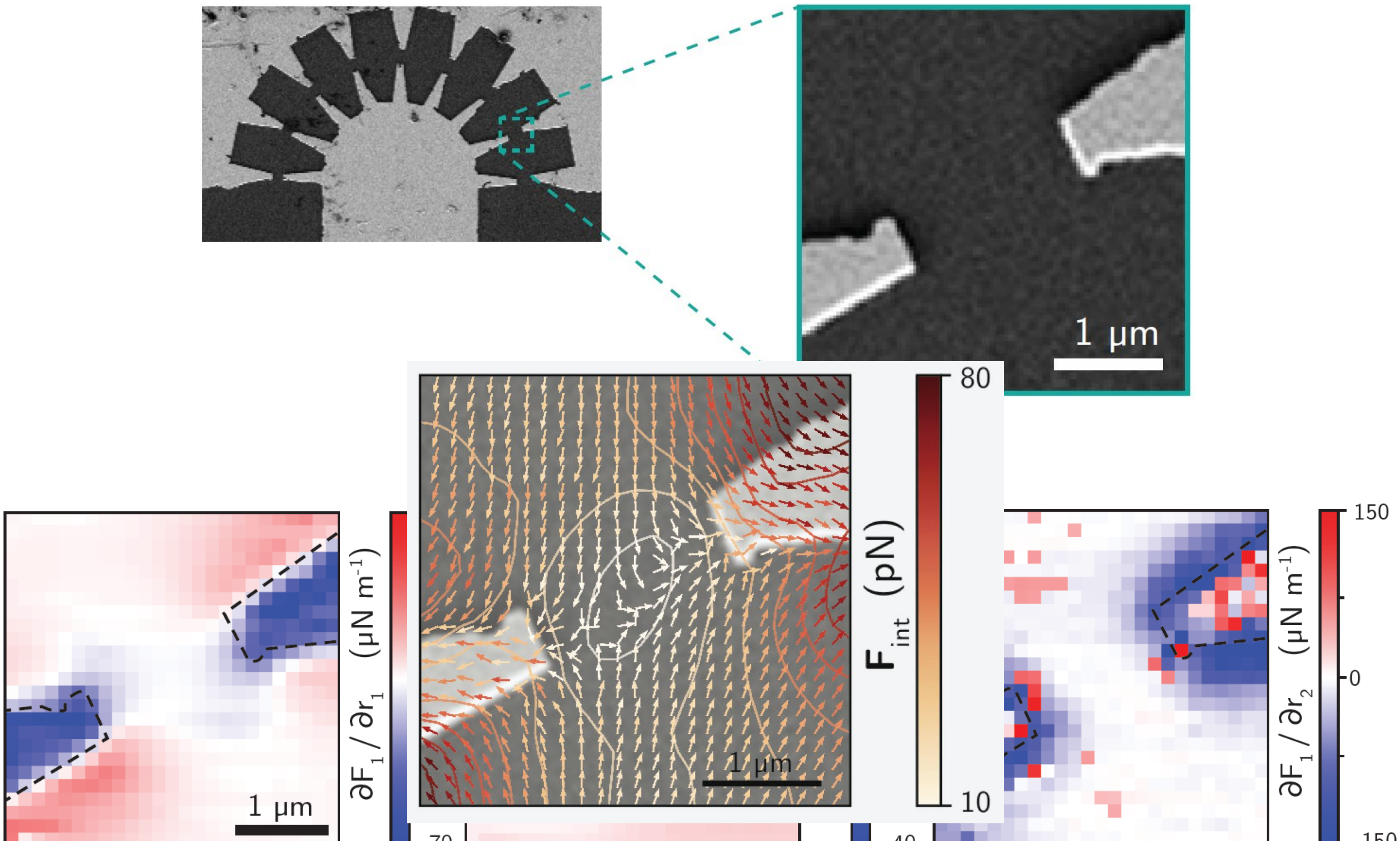
# NW scanning force microscopy



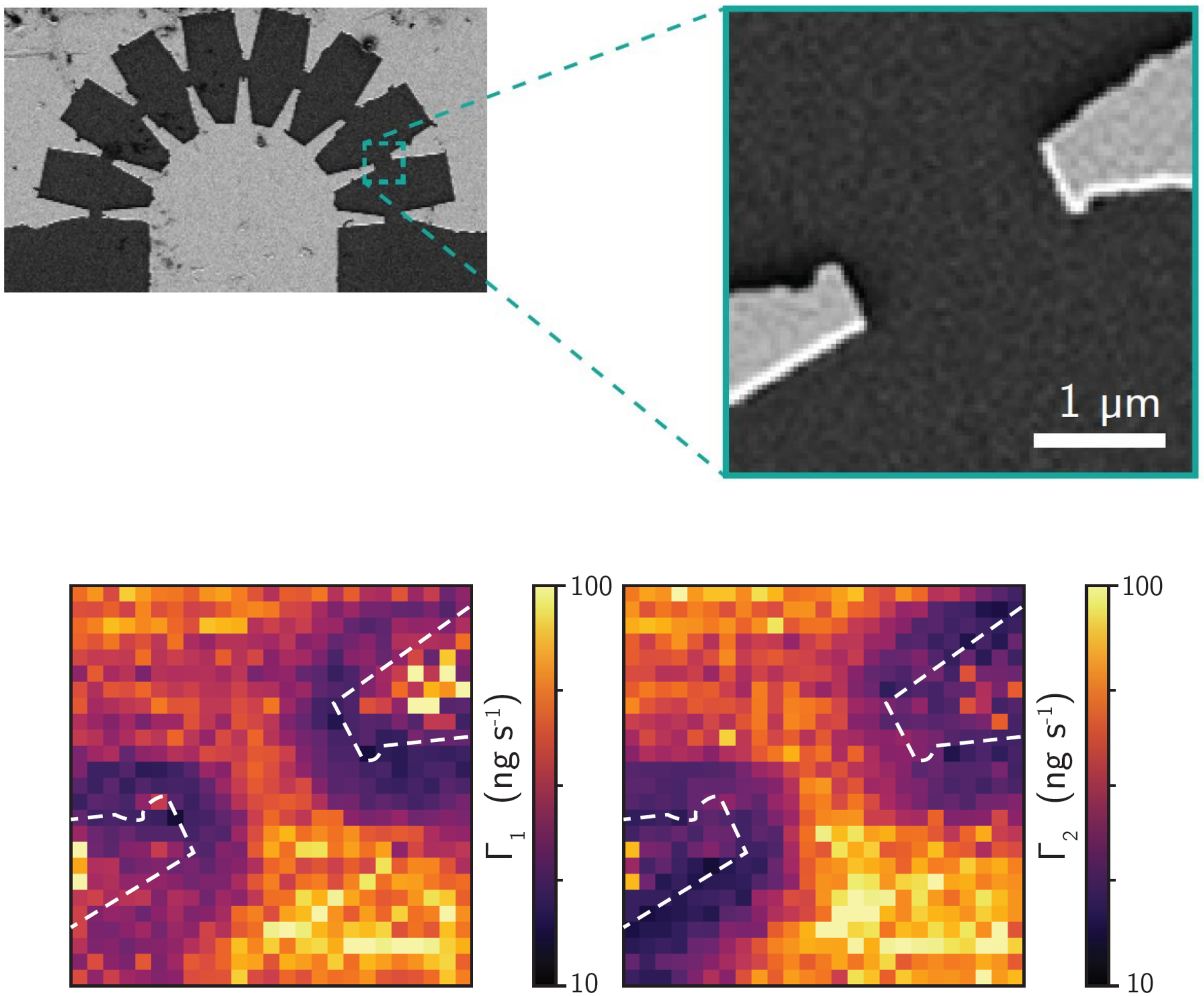

N. Rossi et al., *Nat. Nanotechnol.* **12**, 150 (2017).

# NW scanning force microscopy



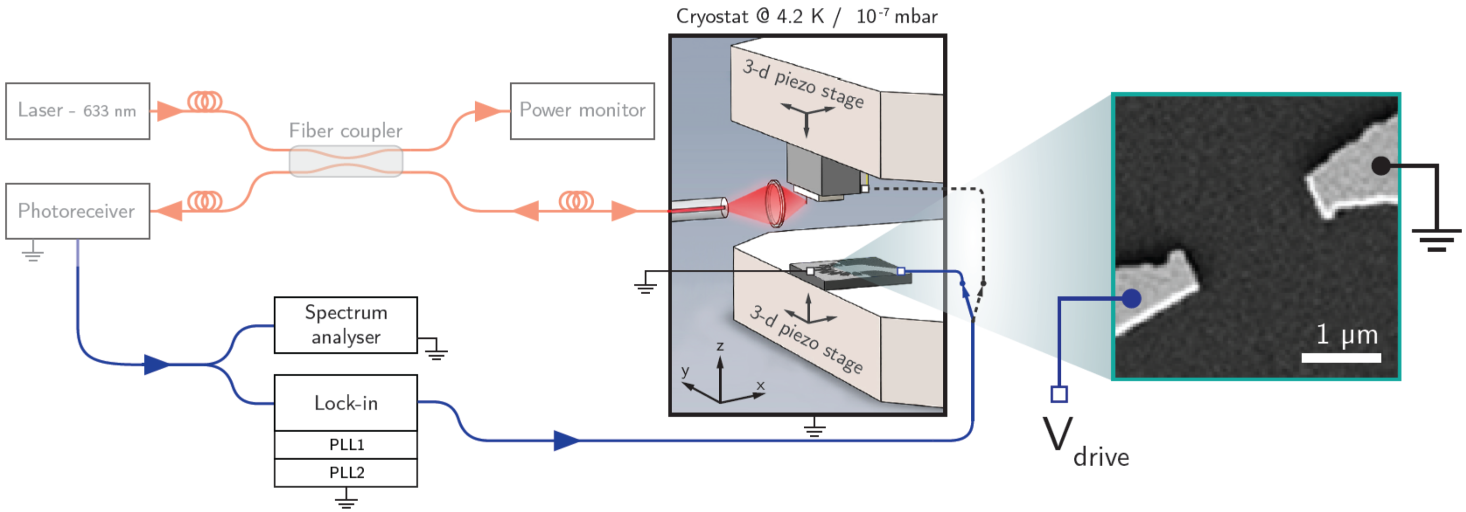

N. Rossi et al., *Nat. Nanotechnol.* **12**, 150 (2017).

# NW scanning force microscopy: Friction




N. Rossi et al., *Nat. Nanotechnol.* **12**, 150 (2017).

# Driving with AC voltage




**Electrostatic tip-sample forces**

- Charge $\mathbf{F}_q = q\mathbf{E}$ $V_{drive} = V_q sin(\omega_0 t)$ $V_q = 2mV$
- Polarizability $\mathbf{F}_p = -\nabla(\alpha|\mathbf{E}|^2)$ $V_{drive} = V_p sin(\frac{\omega_0}{2} t)$ $V_p = 20mV$

N. Rossi et al., *Nat. Nanotechnol.* **12**, 150 (2017).

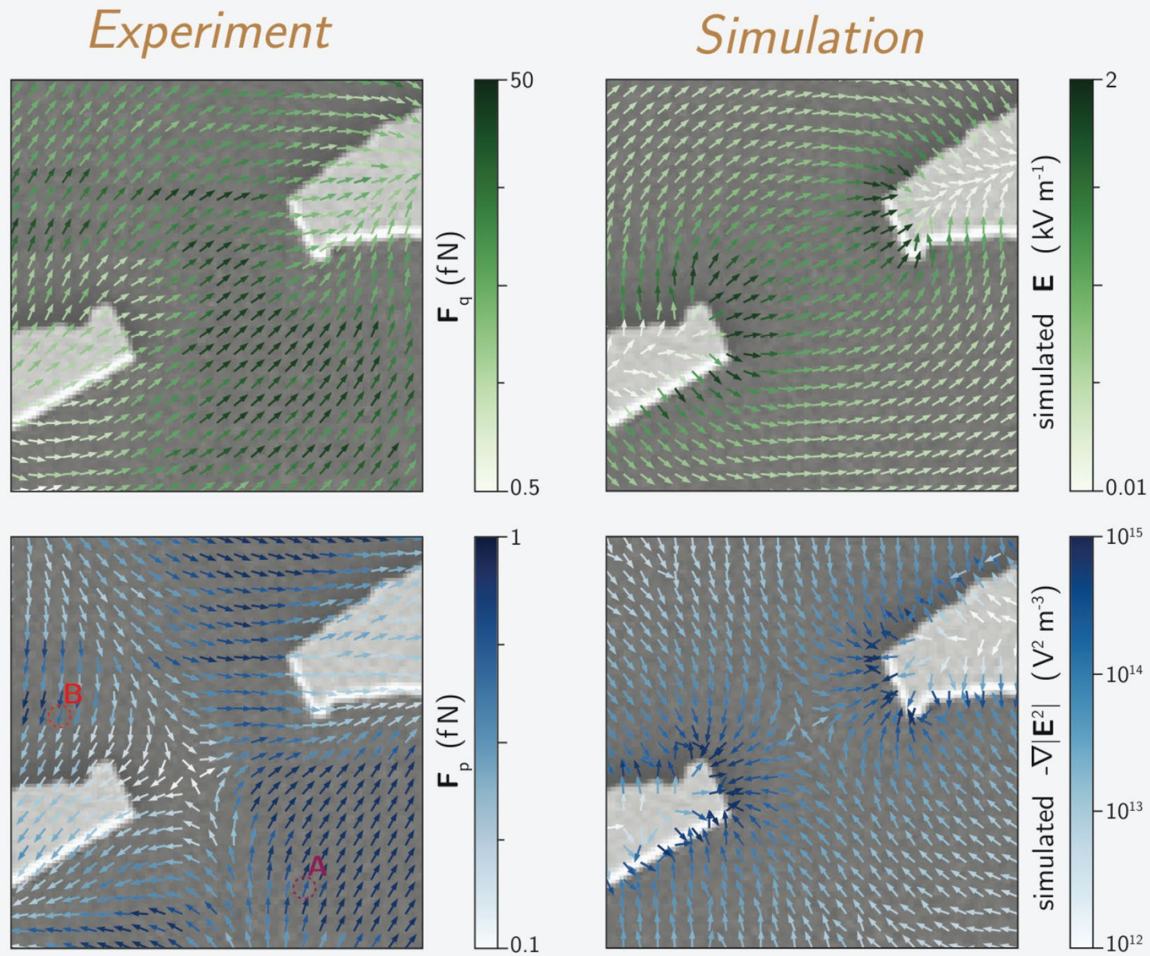
# AC force fields

*Electrostatic force*

- $\mathbf{F}_q = q\mathbf{E}$
- $V_{drive}$ sweep through $f_{1,2}$
- $q_{tot} = 30 \pm 10e$

*Dielectric force*

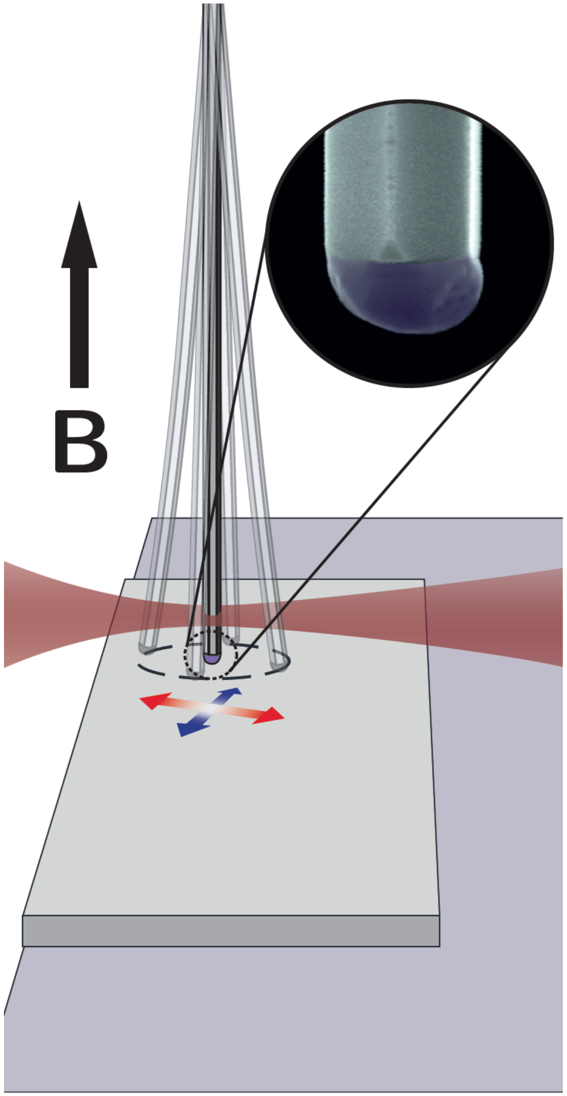
- $\mathbf{F}_p = -\nabla(\alpha|\mathbf{E}|^2)$
- $V_{drive}$ sweep through $\frac{f_{1,2}}{2}$
- $\alpha = 10^{-29} \frac{C}{Vm}$

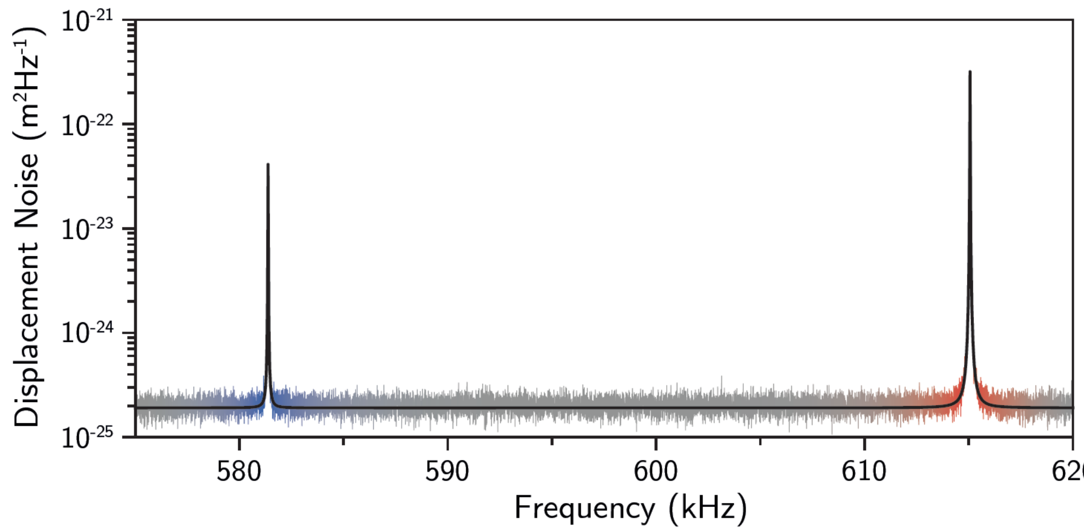



N. Rossi et al., *Nat. Nanotechnol.* **12**, 150 (2017).

See also: Mercier de Lépinay et al., *Nat. Nanotechnol.* **12**, 156 (2017).

# NWs with magnetic tips



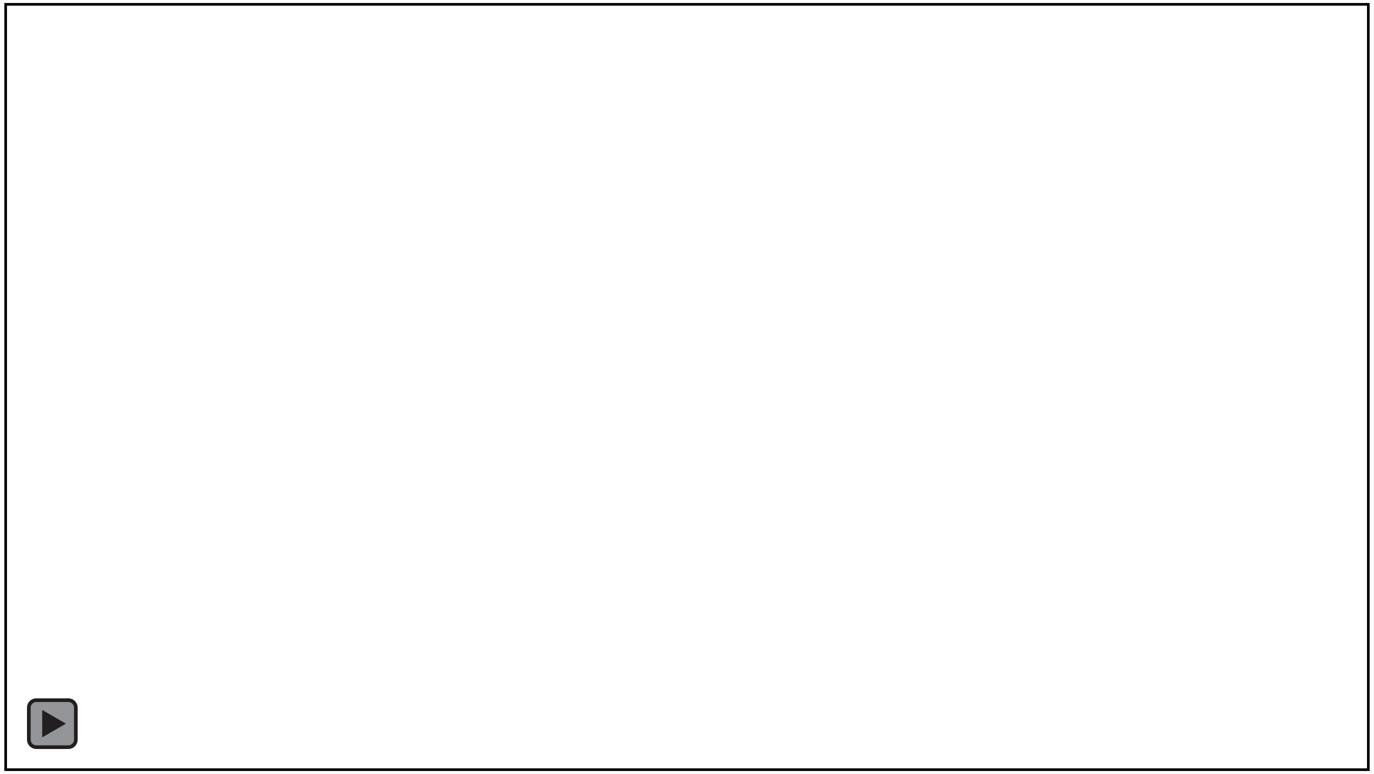




Length $\sim 17 \pm 1\,\mu$m with diameter $\sim 225 \pm 15$ nm

- $f \sim [500 - 700]$ kHz
- $M \sim 700$ fg
- $Q \sim [30 - 50] \times 10^3$
- $\Gamma \sim 50 \times 10^{-15}$ kg/s
- $k \sim [1 - 10]$ mN/m
- $F_{min} \sim 4$ aN/$\sqrt{\text{Hz}}$

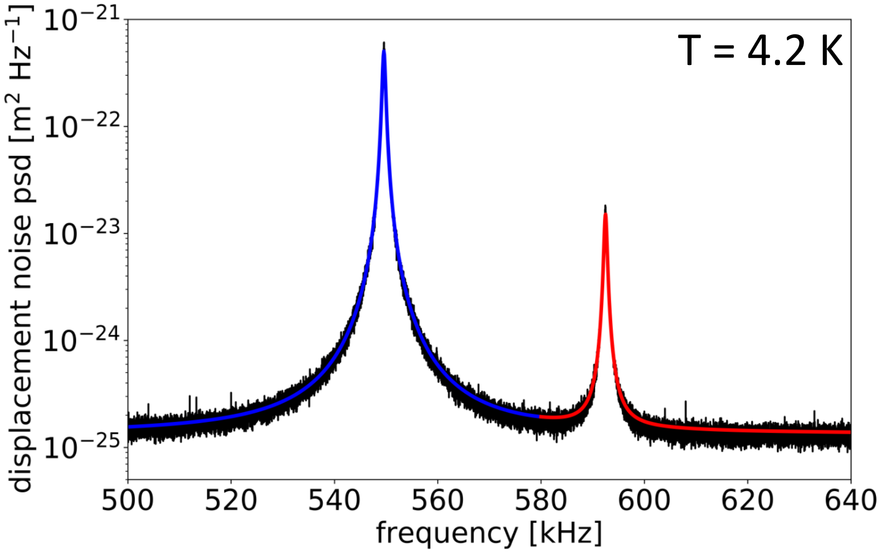
Rossi et al., *Nano Lett.* **19**, 930 (2019).

# Fully magnetic NWs



De Teresa et al., *J. Phys. D: Appl. Phys.* **49**, 243003 (2016).

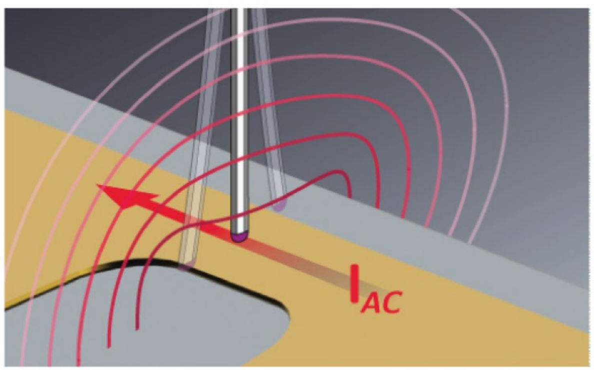
# Fully magnetic NWs

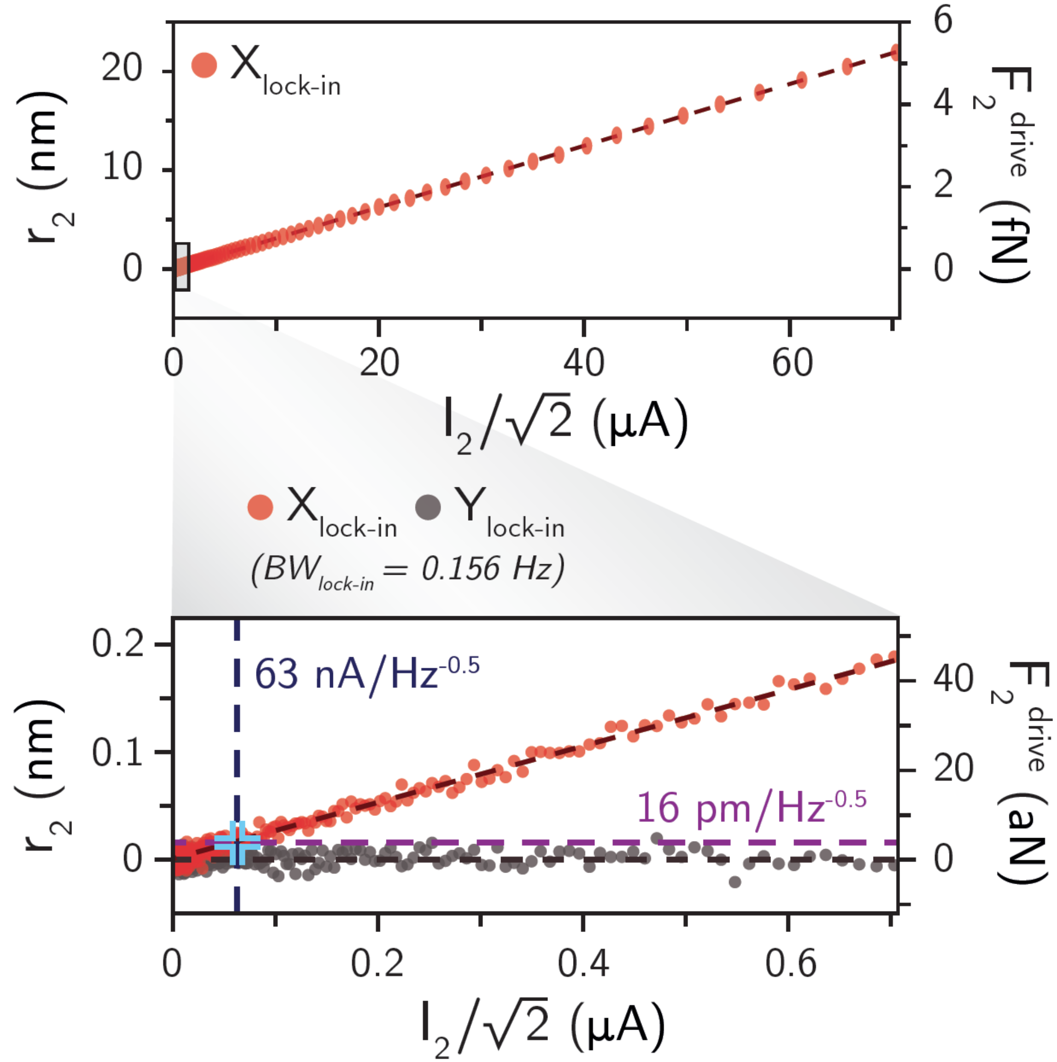




- 10 µm length, 100 nm diameter
- $Q \approx 2000$
- $k \approx 1$ mN/m
- $m_{\text{eff}} \approx 200$ fg
- $F_{\text{min}} \approx 25$ aN/Hz$^{1/2}$

Mattiat et al., *Phys. Rev. Appl.* **13**, 044043 (2020).

# Quantifying sensitivity

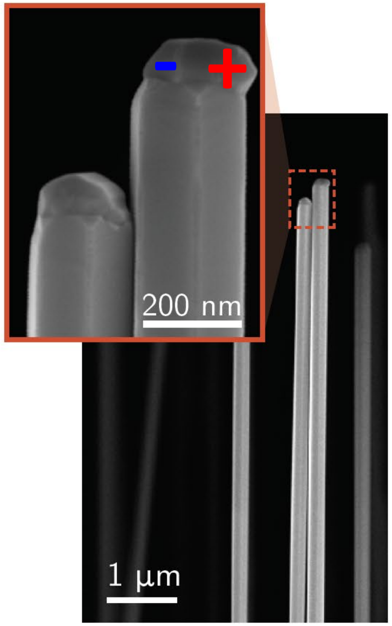



Rossi et al., *Nano Lett.* **19**, 930 (2019).

# Quantifying sensitivity

MBE-grown MnAs-tipped NWs




$F_{min} = 4\ aN/(Hz)^{1/2}$

At 250 nm spacing:

$dB/dx_{min} = 11\ mT/m(Hz)^{1/2}$

FEBID-grown Co NWs


$F_{min} = 25\ aN/(Hz)^{1/2}$

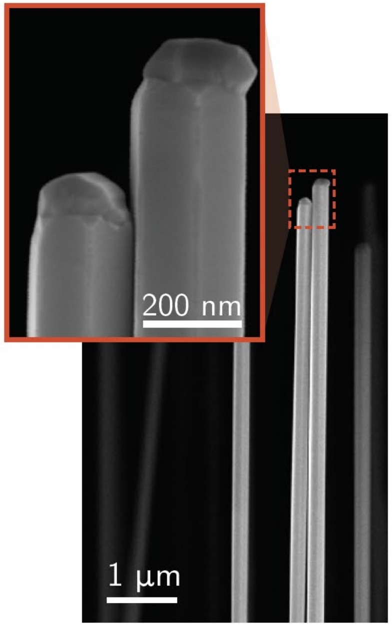
At 200 nm spacing:

$B_{min} = 3\ nT/(Hz)^{1/2}$

Rossi et al., *Nano Lett.* **19**, 930 (2019).
Mattiat et al., *Phys. Rev. Appl.* **13**, 044043 (2020).

# Quantifying sensitivity

MBE-grown MnAs-tipped NWs




250 nm tip diameter

<u>At 250 nm spacing:</u>

$M_{min} = 50\ \mu_B/(\mathrm{Hz})^{1/2}$

$\Phi_{min} = 1\ \mu\Phi_0/(\mathrm{Hz})^{1/2}$

$I_{min} = 10\ \mathrm{nA}/(\mathrm{Hz})^{1/2}$

FEBID-grown Co NWs


100 nm tip diameter

<u>At 100 nm spacing:</u>

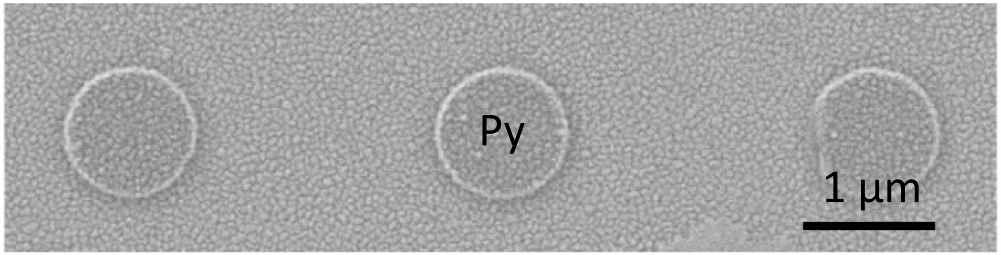
$M_{min} = 60\ \mu_B/(\mathrm{Hz})^{1/2}$

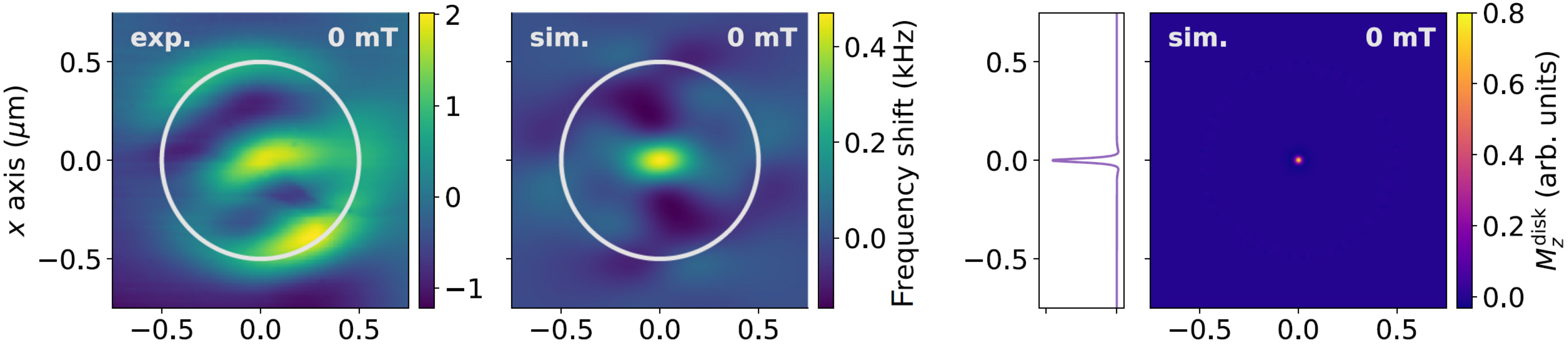
$\Phi_{min} = 6\ \mu\Phi_0/(\mathrm{Hz})^{1/2}$

$I_{min} = 8\ \mathrm{nA}/(\mathrm{Hz})^{1/2}$

Kirtley, *Rep. Prog. Phys.* **73**, 126501 (2010)

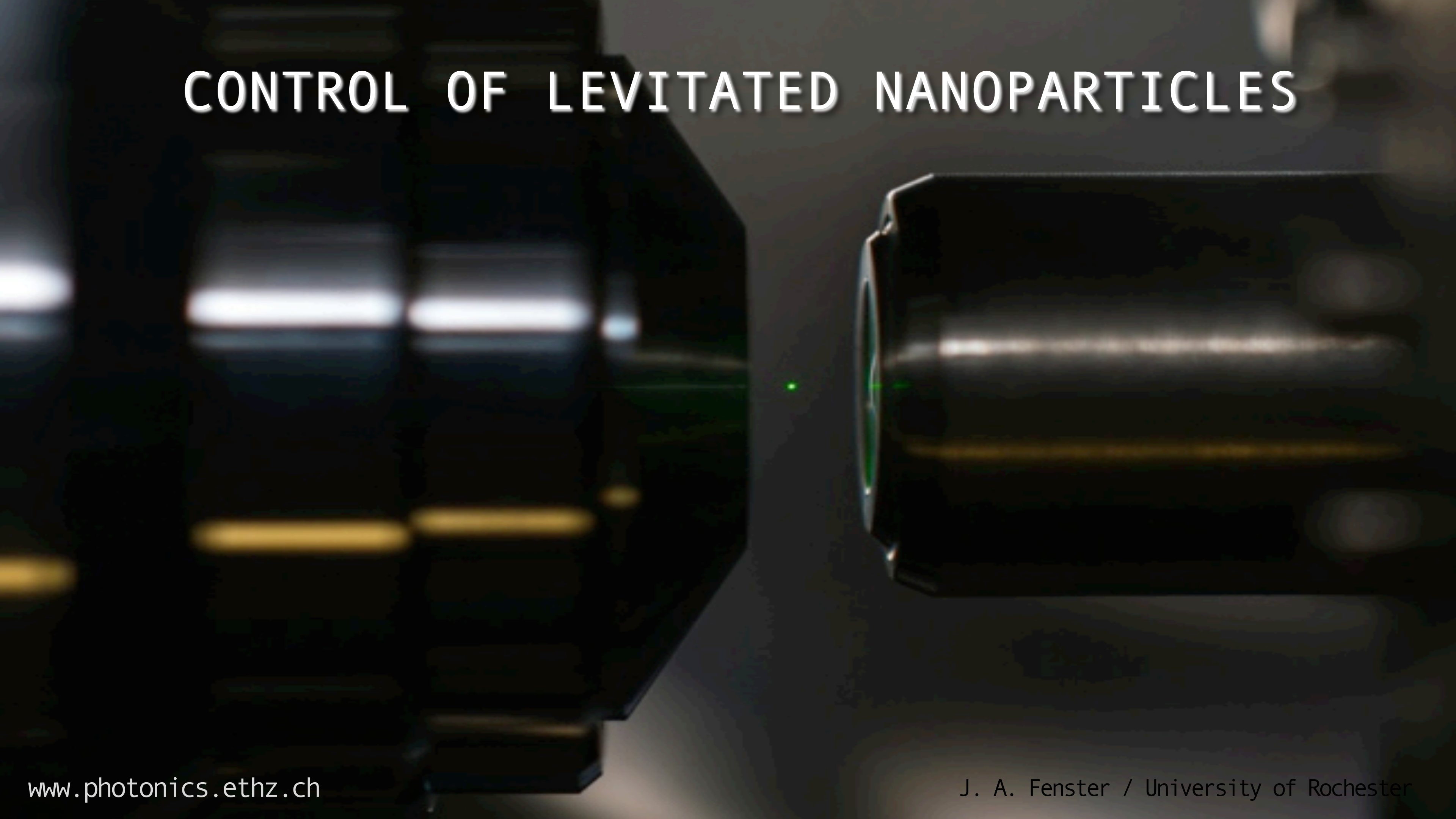
# NW MFM




Mattiat et al., *Phys. Rev. Appl.* **13**, 044043 (2020).

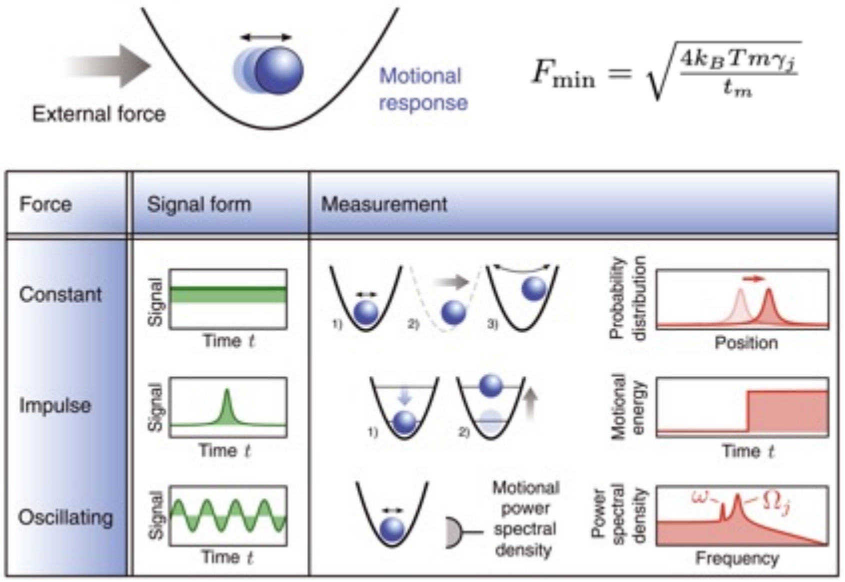
# CONTROL OF LEVITATED NANOPARTICLES



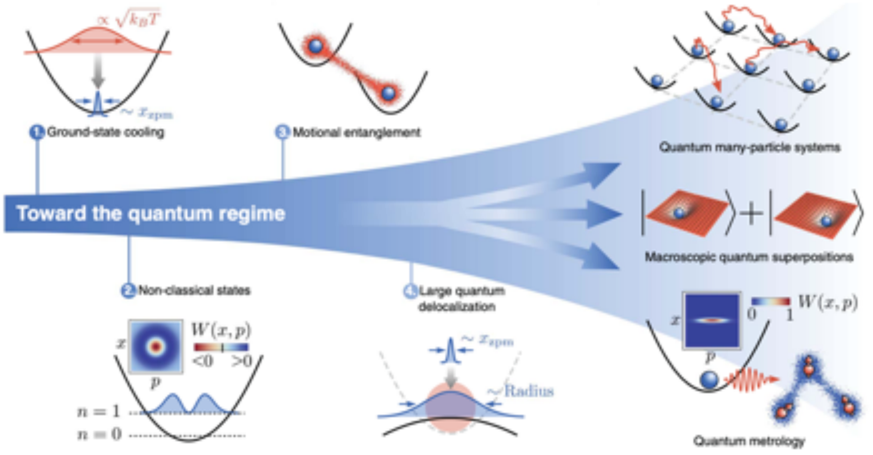
J. A. Fenster / University of Rochester

# MOTIVATION

## Sensing




## Fundamental physics




Gonzalez-Ballestero *et al*, **Science** 374, 6564 (2021)

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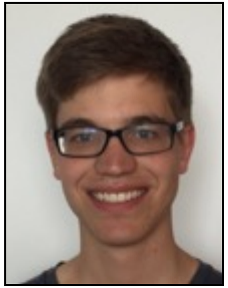
HU:



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SNB:

Dominik Windey

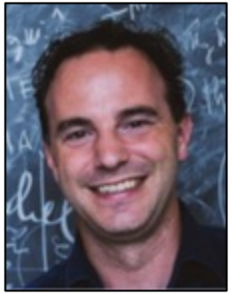
ETH:

Romain Quidant

IQOQI:

Oriol Romero-Isart

U.Vienna:



Markus Aspelmeyer

U.Innsbruck:

Tracy Northup

ETH zürich

# OUTLINE

1: INTRODUCTION (A PERSONAL STORY)

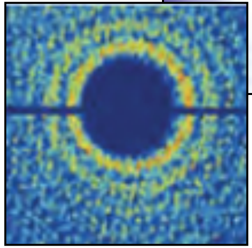
2: Q-XTREME & GROUND-STATE

3: STATE EXPANSION

4: OUTLOOK

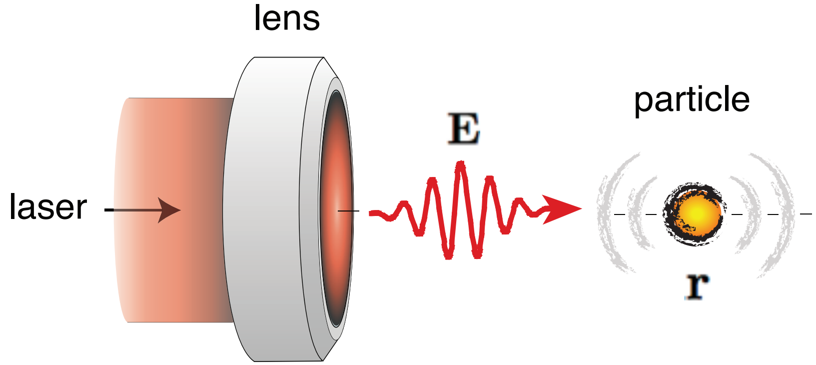
SLAC NATIONAL ACCELERATOR LABORATORY

# LINAC Coherent Light Source (LCLS)



www.slac.stanford.edu

# CONTROL BY OPTICAL FORCES




$$m\ddot{\mathbf{r}} = \langle \mathbf{F} \rangle$$

$$\rightarrow \frac{\alpha'}{2} \underbrace{\sum_{i=x,y,z} \mathrm{Re}\{E_i^* \nabla E_i\}}_{\nabla [\mathbf{E}^* \cdot \mathbf{E}]} + \frac{\alpha''}{2} \sum_{i=x,y,z} \mathrm{Im}\{E_i^* \nabla E_i\}$$

*c.f.* Principles of Nano-Optics (Cambridge)

letters to nature

# Cavity cooling of a microlever

**Constanze Höhberger Metzger & Khaled Karrai**

*Center for NanoScience and Sektion Physik, Ludwig-Maximilians-Universität, Geschwister-Scholl-Platz 1, 80539 München, Germany*

1002 NATURE | VOL 432 | 23/30 DECEMBER 2004 | www.nature.com/nature

$$m\frac{d^2z}{dt^2} + m\Gamma\frac{dz}{dt} + Kz = F_{th} + \sum_n \int_0^t \frac{dF_n[z(t')]}{dt} h_n(t - t')dt'$$

As we will see below, the essence of cooling is based on the fact that the optically induced forces acting on the lever are delayed with respect to a sudden change in the lever position.

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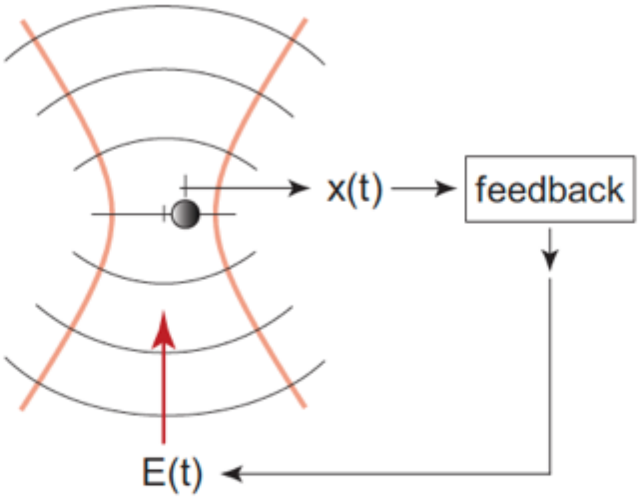
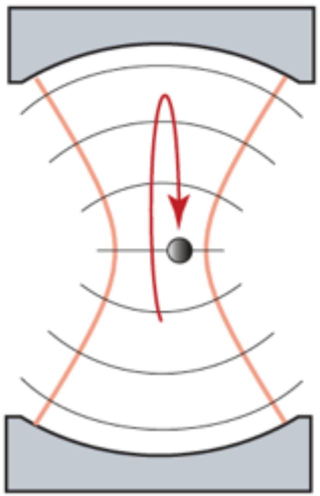
# FEEDBACK CONTROL

Active Feedback:




Passive Feedback:



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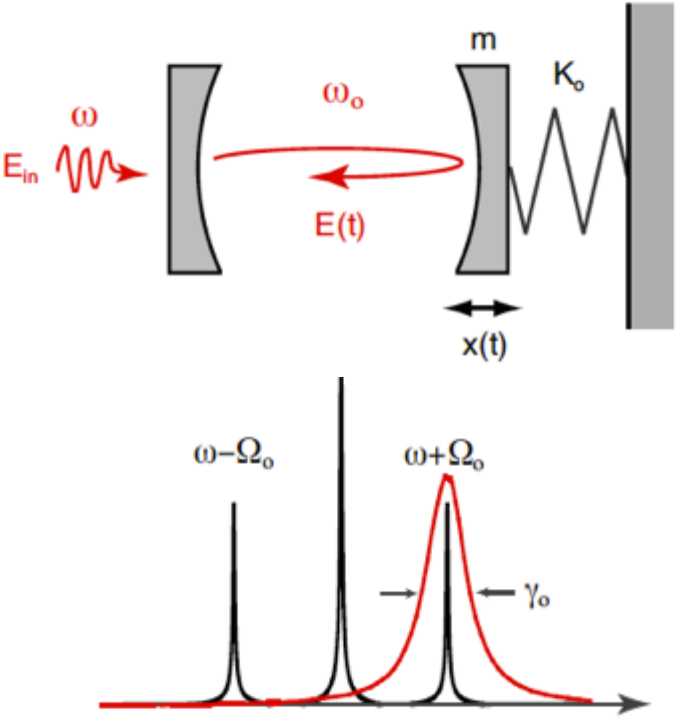
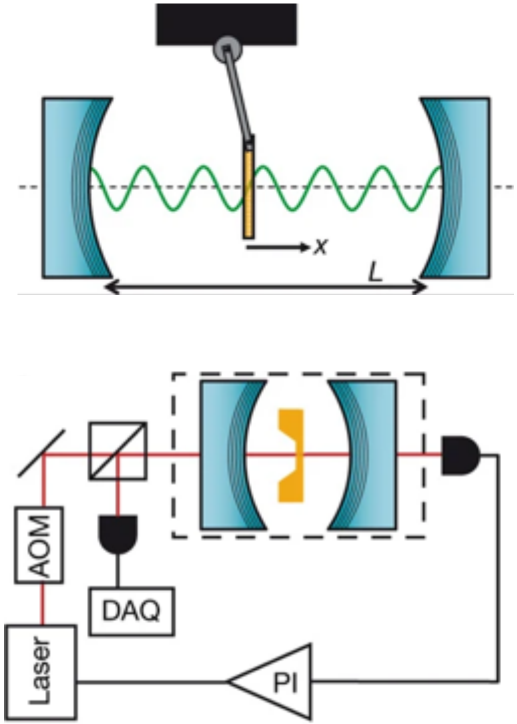
# CAVITY OPTOMECHANICS

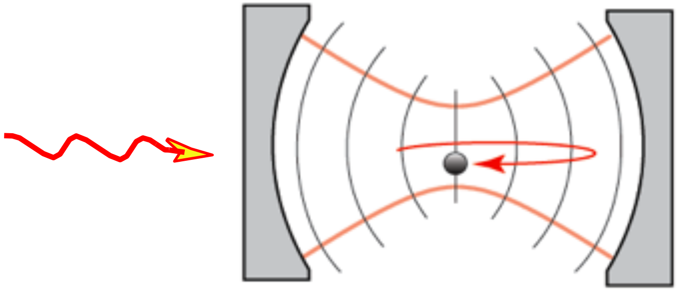



Aspelmeyer, Kippenberg & Marquardt, *Rev. Mod. Phys.* **86**, 1391 (2014).

J.D. Thompson *et al.*, *Nature* ***452***, 72 (2008).

# LEVITATED CAVITY OPTOMECHANICS



PNAS

## Cavity opto-mechanics using an optically levitated nanosphere

D. E. Chang[a], C. A. Regal[b], S. B. Papp[b], D. J. Wilson[b], J. Ye[b,c], O. Painter[d], H. J. Kimble[b,1], and P. Zoller[b,e]

PNAS | January 19, 2010 | vol. 107 | no. 3 | 1005–1010

New Journal of Physics
The open-access journal for physics

## Toward quantum superposition of living organisms

Oriol Romero-Isart[1,4], Mathieu L Juan[2], Romain Quidant[2,3] and J Ignacio Cirac[1]

*New Journal of Physics* **12** (2010) 033015

PHYSICAL REVIEW A **81**, 023826 (2010)

## Cavity cooling of an optically trapped nanoparticle

P. F. Barker
*Department of Physics and Astronomy, University College London, WC1E 6BT, United Kingdom*

M. N. Shneider
*Applied Physics Group, Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, New Jersey 08544, USA*

# Cavity cooling of an optically levitated submicron particle

Nikolai Kiesel[1,2], Florian Blaser[1], Uroš Delić, David Grass, Rainer Kaltenbaek, and Markus Aspelmeyer[2]

Vienna Center for Quantum Science and Technology (VCQ), Faculty of Physics, University of Vienna, A-1090 Vienna, Austria

14180–14185 | PNAS | August 27, 2013 | vol. 110 | no. 35

The damping $\gamma_0$ of the mechanical resonator is dominated by the ambient pressure of the background gas down to a few millibars (Fig. 2*B*). Below these pressures, the submicron particle is not stably trapped anymore, whereas trapping times up to several hours can be achieved at a pressure of a few millibars. This is a known, yet unexplained phenomenon (17, 18, 44).

333 Applied Physics Letters, Vol. 28, No. 6, 15 March 1976

# Optical levitation in high vacuum

A. Ashkin and J. M. Dziedzic

*Bell Telephone Laboratories, Holmdel, New Jersey 07733*
(Received 17 November 1975)

Optical levitation of highly transparent particles has been observed in the high-vacuum regime where viscous damping and thermal conductivity are small, the particle is cooled only by thermal radiation, and radiometric forces are negligible. The effects of an impulse and adiabatic manipulation on the dynamics of a sphere were studied from atmospheric pressure down to $\sim 10^{-6}$ Torr. The calculated time for an oscillating particle to decay to half-amplitude due to the intrinsic optical damping at zero pressure is $\sim 0.7$ years.

202 Applied Physics Letters, Vol. 30, No. 4, 15 February 1977

# Feedback stabilization of optically levitated particles

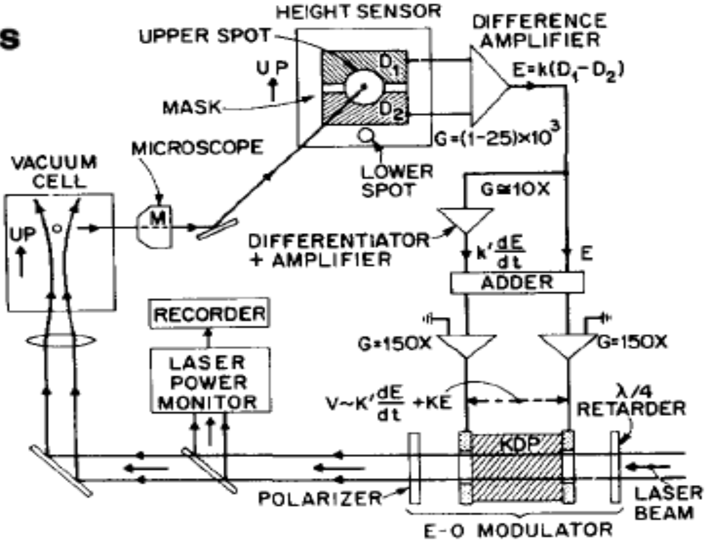
A. Ashkin and J. M. Dziedzic

*Bell Telephone Laboratories, Holmdel, New Jersey 07733*
(Received 5 November 1976)

We demonstrate the locking of an optically levitated sphere to an external reference using an electronic feedback system. This provides a new external source of damping for the stabilization and manipulation of particles in vacuum and at atmospheric pressure. The method permits accurate and continuous monitoring of applied forces. Numerous applications are suggested.

It is of interest to consider the intrinsic damping of an oscillating levitated particle in the limit of total vacuum, i.e., zero viscous damping. Consider first a

Particles ~10 μm in size



VOLUME 83, NUMBER 16 PHYSICAL REVIEW LETTERS 18 OCTOBER 1999

**Cooling of a Mirror by Radiation Pressure**

P. F. Cohadon,* A. Heidmann,† and M. Pinard‡
*Laboratoire Kastler Brossel,§ Case 74, 4 place Jussieu, F75252 Paris Cedex 05, France*
(Received 30 March 1999; revised manuscript received 24 June 1999)

PRL **99,** 017201 (2007) PHYSICAL REVIEW LETTERS week ending 6 JULY 2007

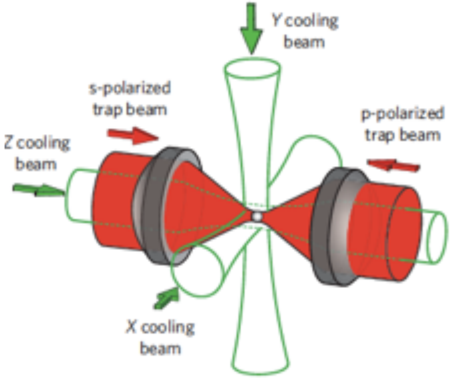
**Feedback Cooling of a Cantilever's Fundamental Mode below 5 mK**

M. Poggio,[1,2] C. L. Degen,[1] H. J. Mamin,[1] and D. Rugar[1]
[1]*IBM Research Division, Almaden Research Center, 650 Harry Rd., San Jose California 95120, USA*
[2]*Center for Probing the Nanoscale, Stanford University, 476 Lomita Hall, Stanford California 94305, USA*
(Received 7 February 2007; published 2 July 2007)

nature physics LETTERS
PUBLISHED ONLINE: 20 MARCH 2011 | DOI: 10.1038/NPHYS1952

**Millikelvin cooling of an optically trapped microsphere in vacuum**

Tongcang Li, Simon Kheifets and Mark G. Raizen*



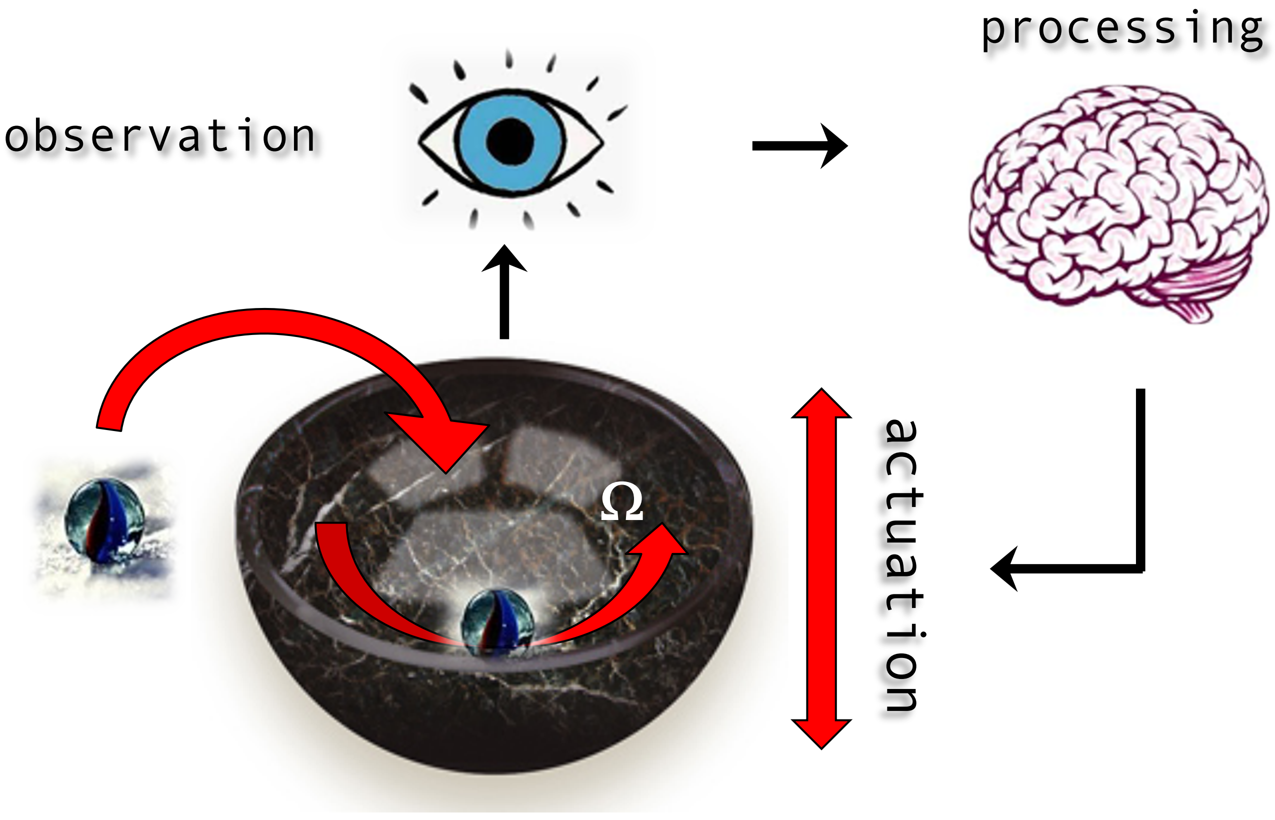

PRL **105,** 173003 (2010) PHYSICAL REVIEW LETTERS week ending 22 OCTOBER 2010

**Feedback Cooling of a Single Neutral Atom**

Markus Koch,[1,*] Christian Sames,[1] Alexander Kubanek,[1] Matthias Apel,[1] Maximilian Balbach,[1] Alexei Ourjoumtsev,[1,†] Pepijn W. H. Pinkse,[2] and Gerhard Rempe[1]

ETH zürich

# FEEDBACK CONTROL



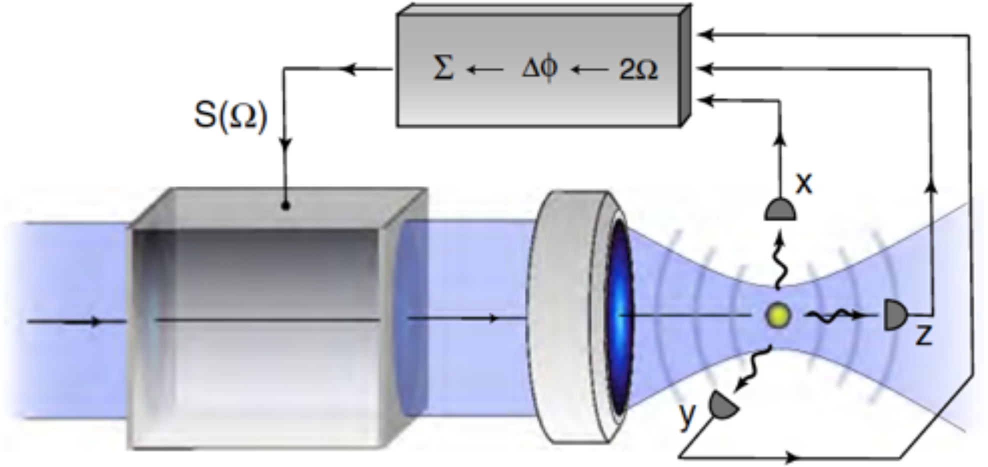
# PARAMETRIC FEEDBACK COOLING

$$\ddot{x}(t) + \gamma\,\dot{x}(t) + \Omega_0^2\,x(t) = (1/m)\left[F_{\mathrm{fluct}}(t) + F_{\mathrm{opt}}(t)\right]$$

$$\rightarrow \Delta k_{\mathrm{trap}}(t)\,x(t)$$

$$\downarrow$$

$$x(t)\dot{x}(t)$$




*PRL* **109**, 103603 (2012)

Parametric feedback (2012) :

PRL 109, 103603 (2012) PHYSICAL REVIEW LETTERS week ending 7 SEPTEMBER 2012

**Subkelvin Parametric Feedback Cooling of a Laser-Trapped Nanoparticle**

Jan Gieseler,[1] Bradley Deutsch,[3] Romain Quidant,[1,2] and Lukas Novotny[3,4]

Backaction limited measurement (2016) :

PRL 116, 243601 (2016) PHYSICAL REVIEW LETTERS week ending 17 JUNE 2016

**Direct Measurement of Photon Recoil from a Levitated Nanoparticle**

Vijay Jain,[1,2] Jan Gieseler,[1] Clemens Moritz,[3] Christoph Dellago,[3] Romain Quidant,[4,5] and Lukas Novotny[1,*]

Optimal control (2019) :

PHYSICAL REVIEW LETTERS 122, 223601 (2019)

**Cold Damping of an Optically Levitated Nanoparticle to Microkelvin Temperatures**

Felix Tebbenjohanns, Martin Frimmer, Andrei Militaru, Vijay Jain, and Lukas Novotny

Optimal detection (2019) :

PHYSICAL REVIEW A 100, 043821 (2019)

**Optimal position detection of a dipolar scatterer in a focused field**

Felix Tebbenjohanns,[*] Martin Frimmer, and Lukas Novotny

Sideband asymmetry (2020) :

PHYSICAL REVIEW LETTERS **124**, 013603 (2020)

**Motional Sideband Asymmetry of a Nanoparticle Optically Levitated in Free Space**

Felix Tebbenjohanns, Martin Frimmer, Vijay Jain, Dominik Windey, and Lukas Novotny

Groundstate (2021):

**Real-time optimal quantum control of mechanical motion at room temperature**

Lorenzo Magrini[1]✉, Philipp Rosenzweig[2], Constanze Bach[1], Andreas Deutschmann-Olek[2], Sebastian G. Hofer[1], Sungkun Hong[3,4], Nikolai Kiesel[1], Andreas Kugi[2,5] & Markus Aspelmeyer[1,6]✉

Nature | Vol 595 | 15 July 2021 | **373**

**Quantum control of a nanoparticle optically levitated in cryogenic free space**

Felix Tebbenjohanns[1,3], M. Luisa Mattana[1,3], Massimiliano Rossi[1,3], Martin Frimmer[1] & Lukas Novotny[1,2]✉

**378** | Nature | Vol 595 | 15 July 2021

# COOLING DYNAMICS

Equation of motion : $\ddot{y} + \gamma\dot{y} + \Omega_0^2 y = \frac{1}{m} F_{\text{fluct}}(t)$

feedback gain

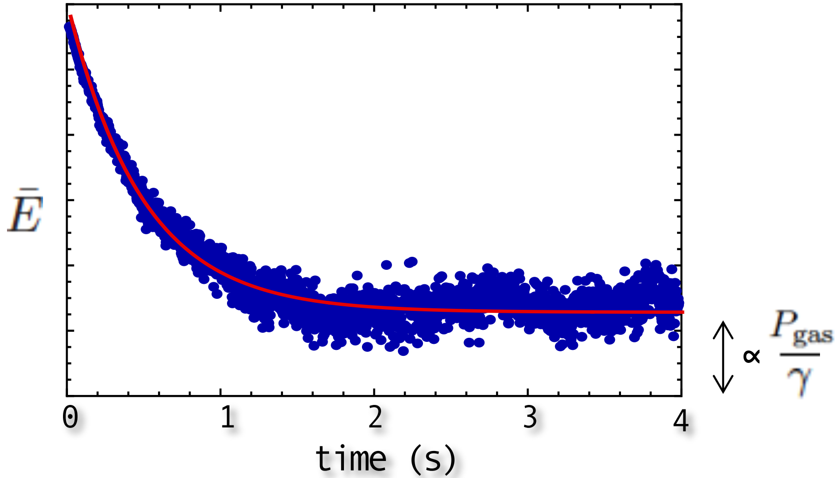
gas pressure

$$S_F(\Omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \langle F_{\text{fluct}}(t)\, F_{\text{fluct}}(t+t')\rangle\, e^{i\Omega t'} dt' \propto P_{\text{gas}}$$

Energy : $\frac{d}{dt}\bar{E}(t) = -\gamma\bar{E}(t) + \frac{\pi}{m} S_F$

cooling heating

$$\rightarrow \quad \bar{E}(t) = E_\infty + [\bar{E}(0) - E_\infty]\, e^{-\gamma t}$$



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# OUTLINE

# GENERATING MACROSCOPIC QUANTUM SUPERPOSITIONS

GROUND-STATE COOLING

↓

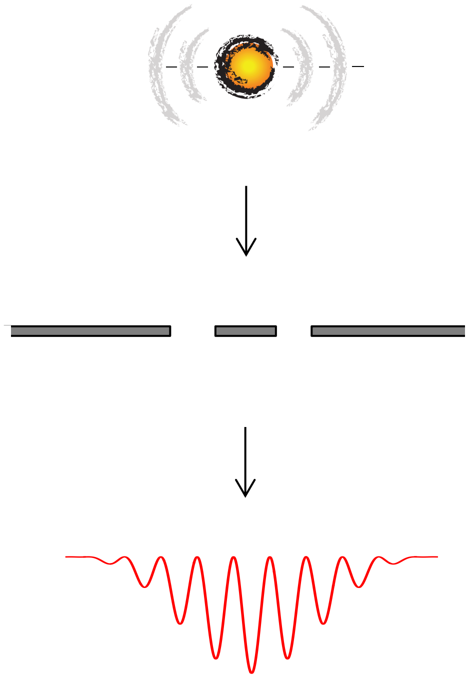
WAVEFUNCTION EXPANSION

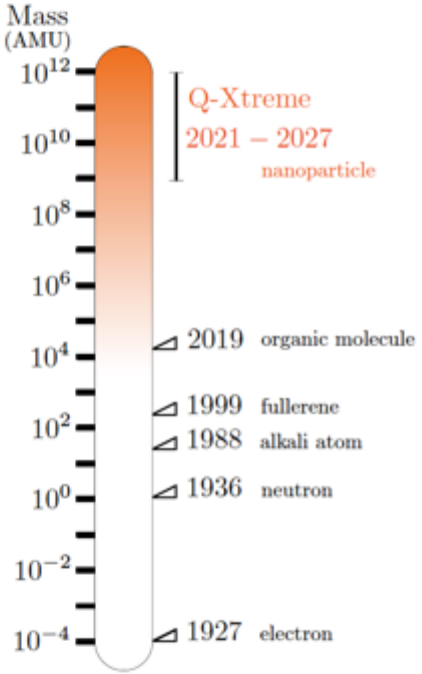
↓

NONLINEAR INTERACTION
(non-Gaussian state)

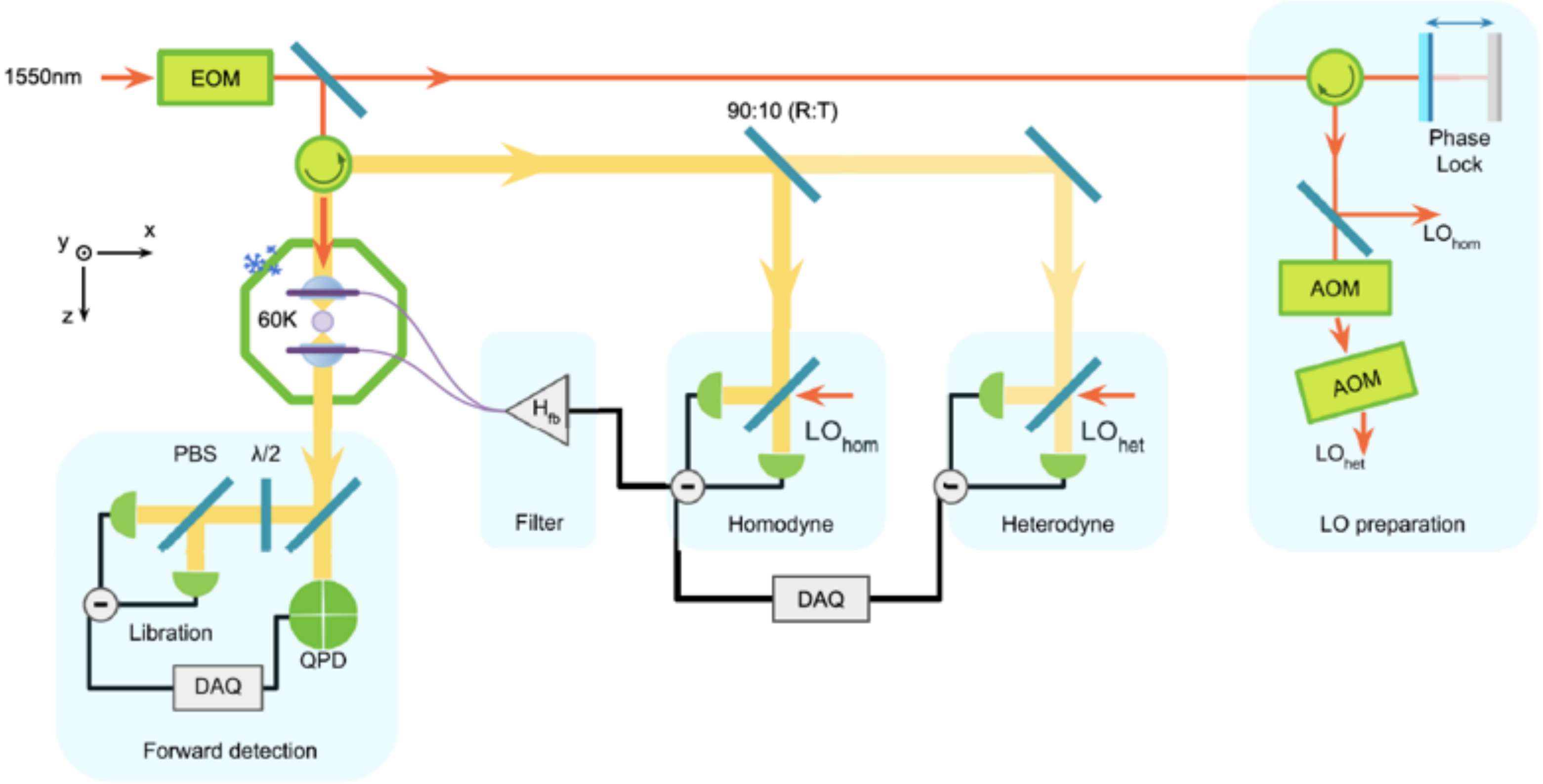
↓

QUANTUM MEASUREMENT





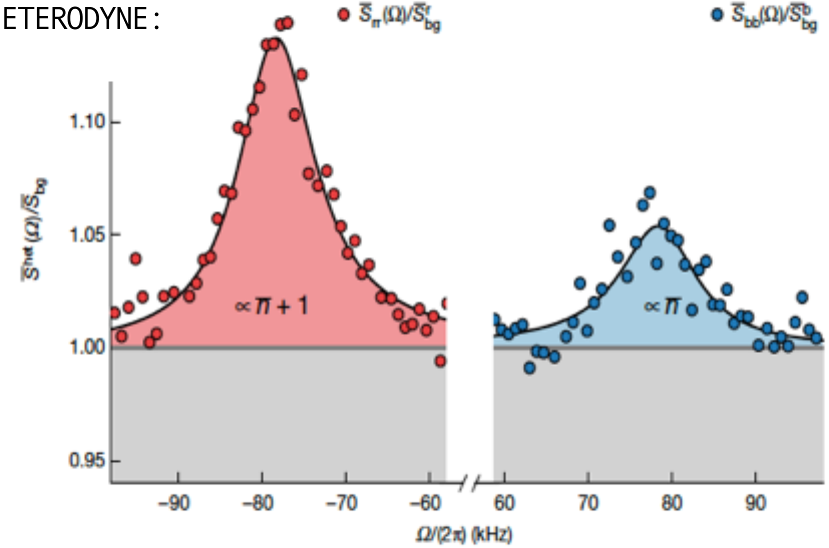
# GROUND-STATE COOLING



*ETH zürich*

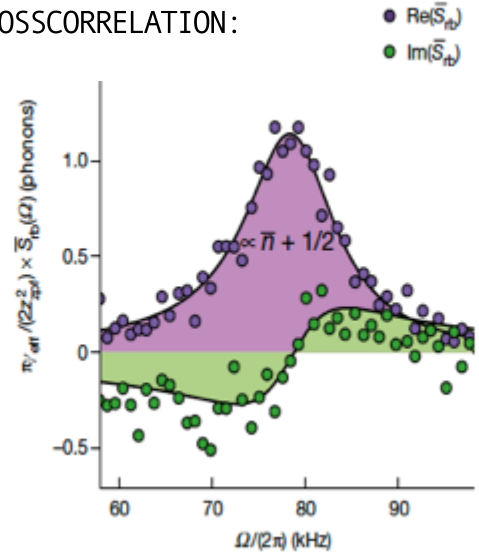
# GROUND-STATE COOLING IN CRYOGENIC FREE SPACE

OUT-OF-LOOP HETERODYNE:




$\bar{n} = 0.66 \pm 0.08$

OUT-OF-LOOP CROSSCORRELATION:




$\bar{n} = 0.64 \pm 0.09$

*Nature* **595**, 378 (2021)

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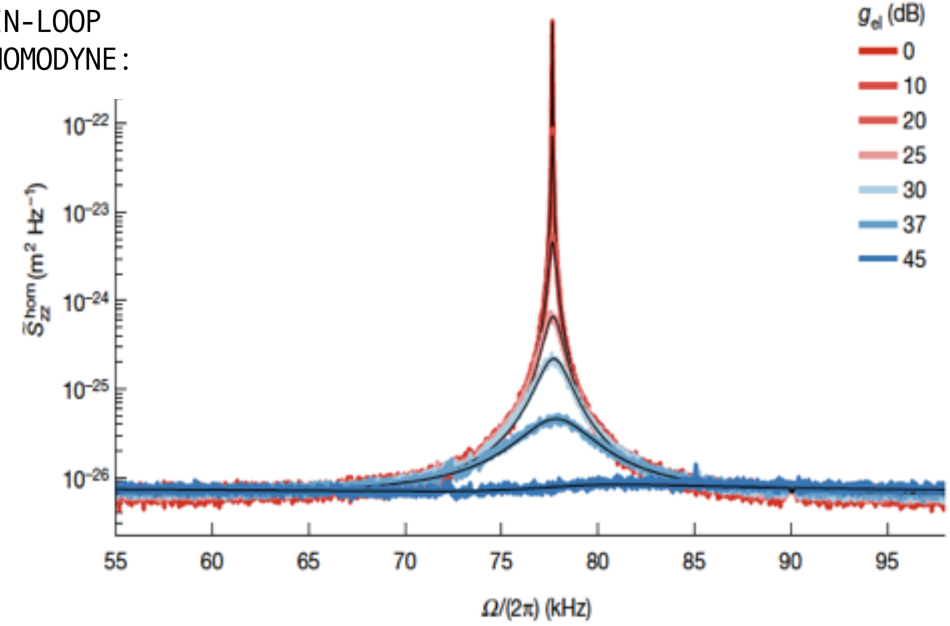
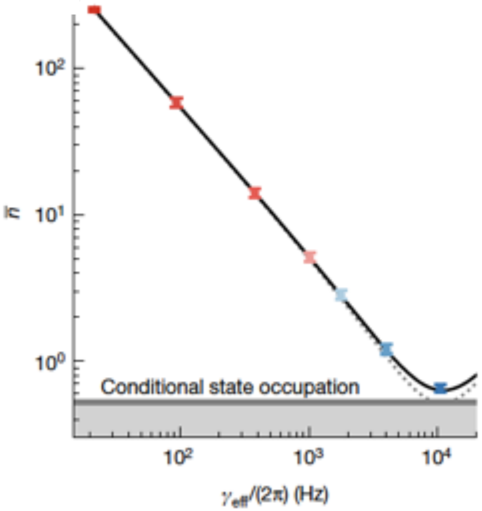
# GROUND-STATE COOLING IN CRYOGENIC FREE SPACE

IN-LOOP HOMODYNE:




$\eta_{meas} = \Gamma_{meas}/\Gamma_{tot} = 0.24 \pm 0.02$




$\bar{n} = 0.65 \pm 0.04$

*Nature* **595**, 378 (2021)

*c.f. Aspelmeyer group* → *Nature* **595**, 373 (2021)

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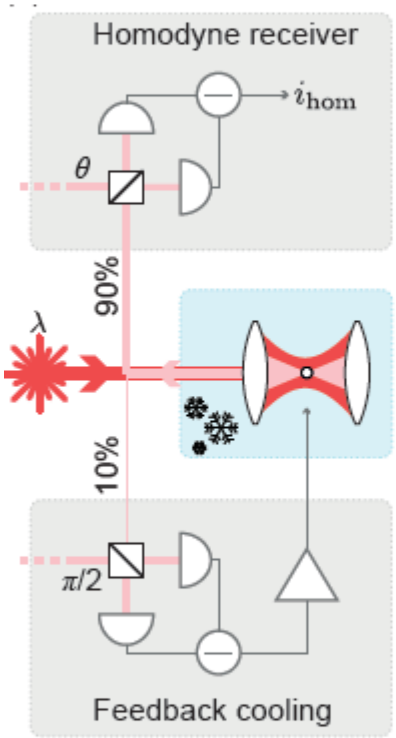
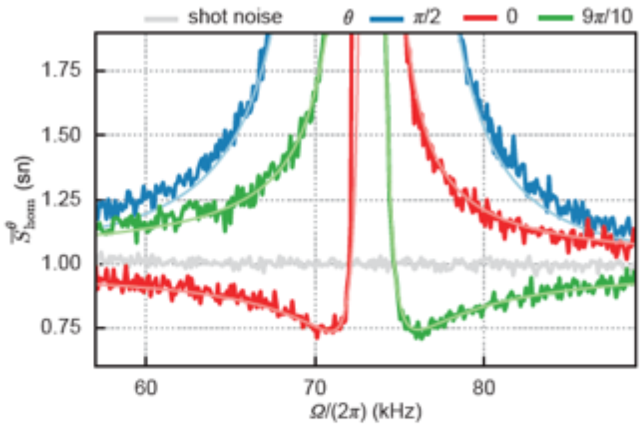
# PONDEROMOTIVE SQUEEZING




$$\bar{S}_{PP} = 1 + \bar{S}^{\theta}_{\mathrm{imp}}\,|m\Omega_0\chi|^2\,\bar{S}^{\mathrm{tot}}_{FF} - 2\,\mathrm{Re}\{(m\Omega_0\chi)\,\bar{S}^{\theta}_{\mathrm{corr}}\}$$

$\downarrow$ $8\Gamma_{\mathrm{meas}}\,(\sin\theta)^2$ — $\downarrow$ $2\Gamma_{\mathrm{tot}}$ — $\downarrow$ $2\Gamma_{\mathrm{meas}}\,\sin(2\theta)$

*PRL* **129**, 053602 (2022)

# OUTLINE

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Q·XTREME Macroscopic Quantum Superpositions

# GENERATING MACROSCOPIC QUANTUM SUPERPOSITIONS

GROUND-STATE COOLING

↓

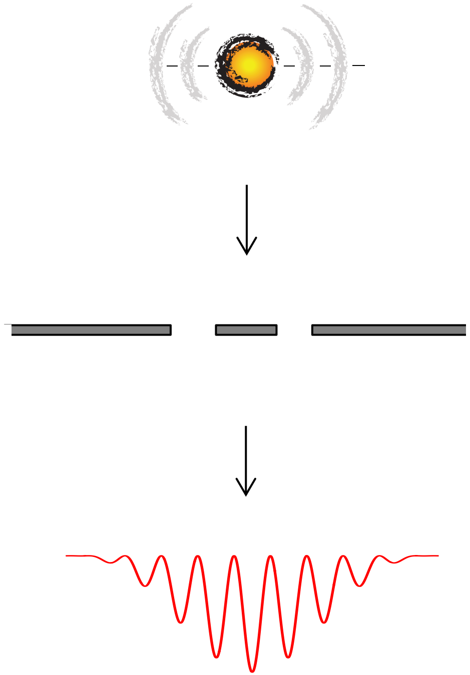
WAVEFUNCTION EXPANSION

↓

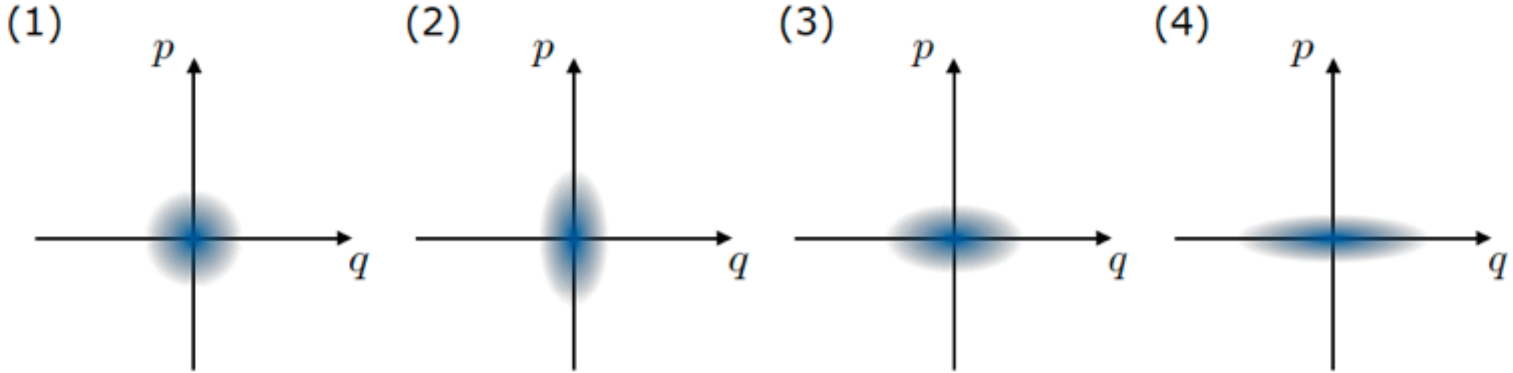
NONLINEAR INTERACTION
(non-Gaussian state)

↓

QUANTUM MEASUREMENT



# WAVEFUNCTION EXPANSION

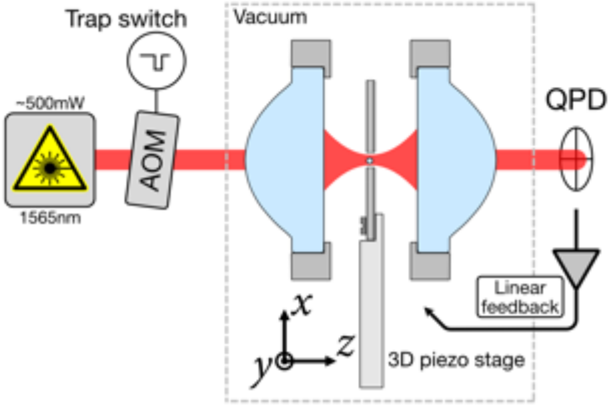


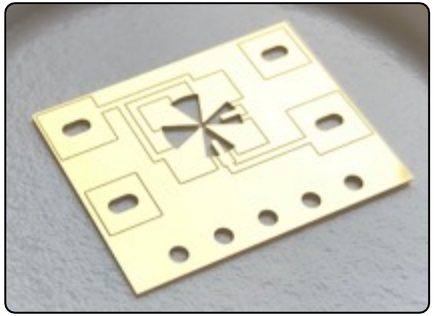

1. Initialization: $|0\rangle$
2. Switch resonance frequency from $\Omega_0$ to $\Omega_1$: $S|0\rangle$
3. Rotate state by a quarter (of the new) period: $\mathrm{e}^{-\frac{\mathrm{i}\pi}{2}b^\dagger b}S|0\rangle$
4. Switch resonance frequency back to $\Omega_0$: $\underbrace{S^\dagger \mathrm{e}^{-\frac{\mathrm{i}\pi}{2}b^\dagger b}S}_{S^2}|0\rangle$

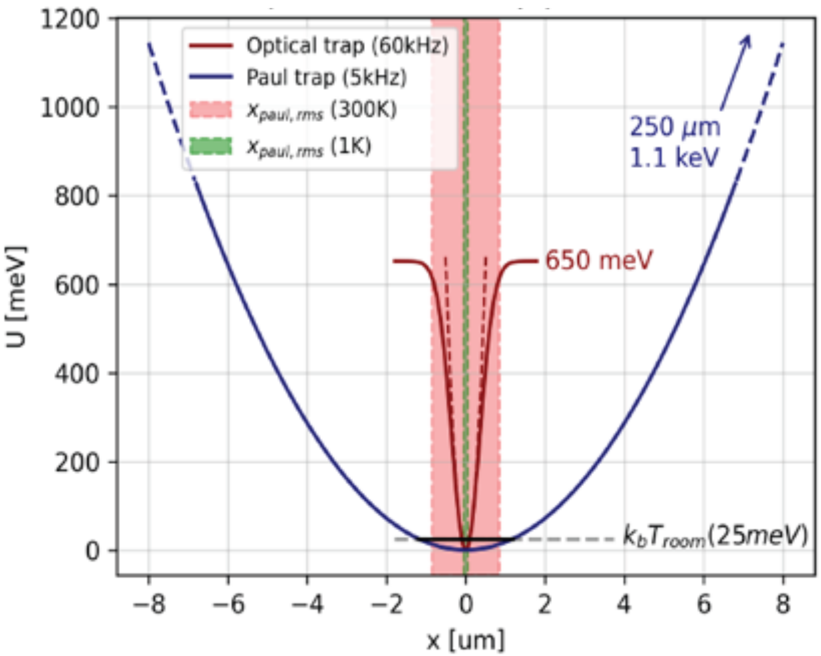
Final squeezing $s^2 = \left(\frac{\Omega_0}{\Omega_1}\right)^2$

J. Janszky et al., *PRA* **39**, 5445 (1989)
M. Rashid et al., *PRL* **117**, 273601 (2016)
M. Xin et al., *PRL* **127**, 183602 (2021)

# HYBRID rf-OPTICAL TRAP






Eric Bonvin

# OUTLINE

1: INTRODUCTION (A PERSONAL STORY)

2: Q-XTREME & GROUND-STATE

3: STATE EXPANSION

4: OUTLOOK

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Q·XTREME Macroscopic Quantum Superpositions

# GENERATING MACROSCOPIC QUANTUM SUPERPOSITIONS

GROUND-STATE COOLING

↓

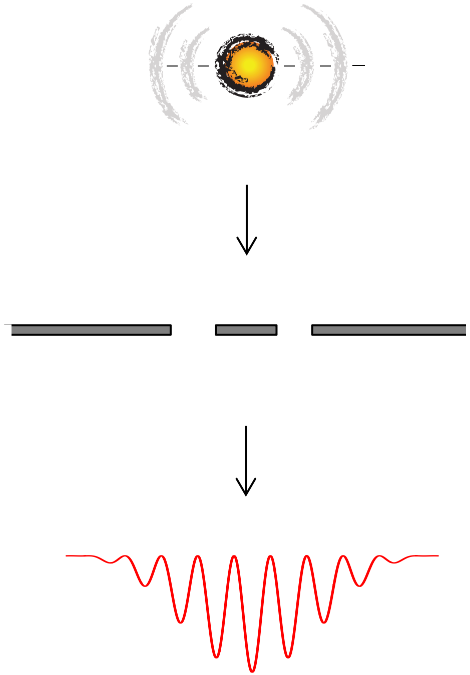
WAVEFUNCTION EXPANSION

↓

NONLINEAR INTERACTION
(non-Gaussian state)

↓

QUANTUM MEASUREMENT



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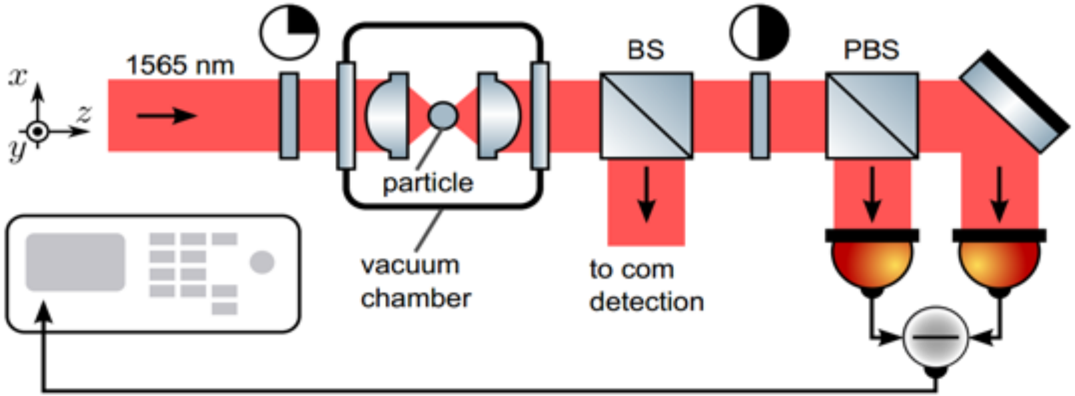
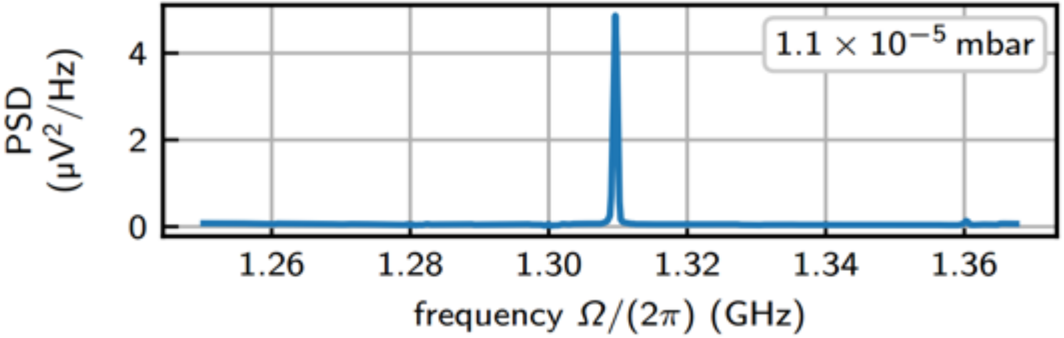
# ROTATIONS




Joanna Zielinska

*c.f.* work by:

M. Arndt
J. Millen
K. Dholakia
P. Zemanek
T. Li
D. Moore
…

*PRL* **121**, 033602 (2018)

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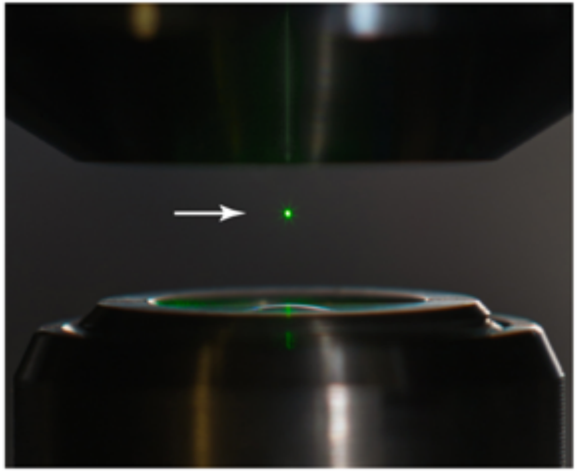
# INERTIAL SENSING

iQLev

European Commission

Horizon 2020
European Union funding
for Research & Innovation

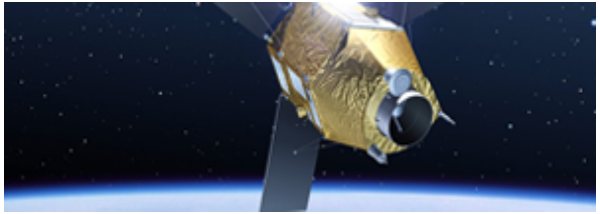


iXblue
industry partcipant

Zurich Instruments
AIRBUS
industry experts

universität wien
universität innsbruck
ICFO The Institute of Photonic Sciences
ETH zürich
scientific participants

iXblue



accelerometers ⟶ ng/Hz$^{1/2}$
gyroscopes ⟶ μdeg/hr

www.photonics.ethz.ch

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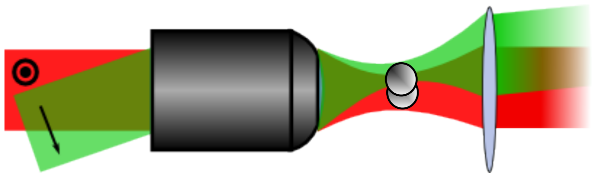
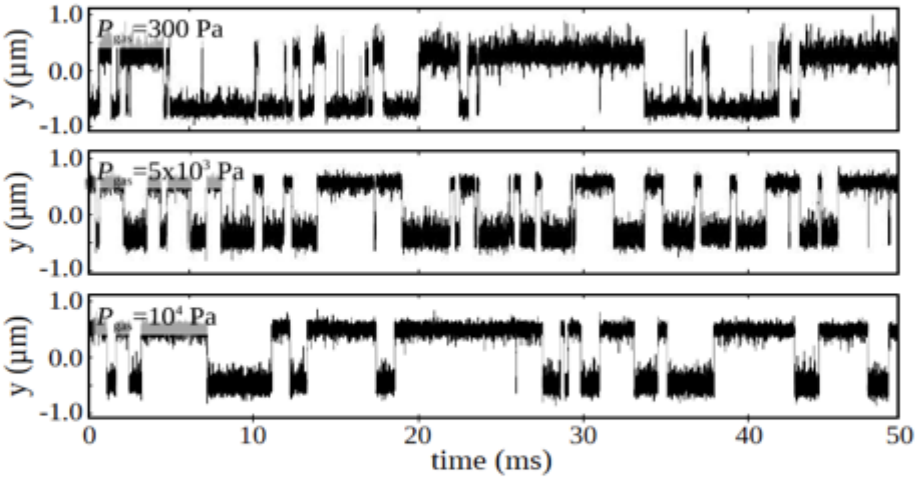
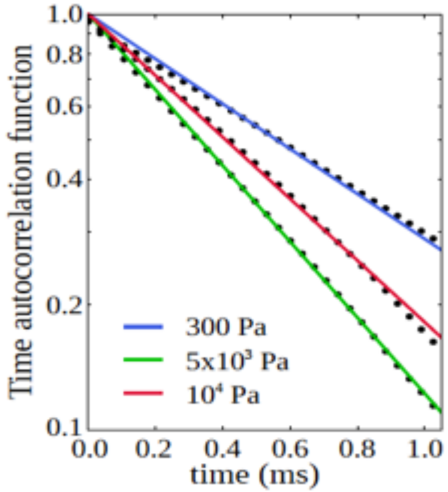
# KINETICS






*Nature Nano* **9**, 358 (2014)

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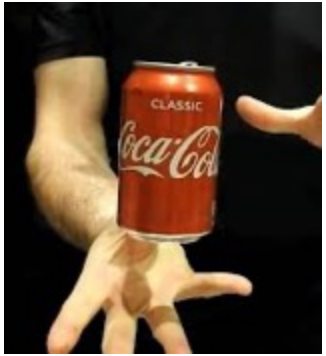
# SIMULATION




$\ddot{x} + \gamma\dot{x} + \Omega_0^2 x = \delta F(t)/m$ (equilibrium dynamics)

$\ddot{x} + \gamma\dot{x} + \Omega_0^2 x[1 + \Omega_0^{-1}\eta x\dot{x}] = \delta F(t)/m$ (parametric feedback)

$\ddot{x} + \gamma\dot{x} + \Omega_0^2 x[1 + \Omega_0^{-1}\eta x\dot{x} + \epsilon\cos(\Omega_m t)] = \delta F(t)/m$ (parametric modulation)

$\ddot{x} + \gamma\dot{x} + \Omega_0^2 x[1 + \Omega_0^{-1}\eta x\dot{x} + \epsilon\cos(\Omega_m t) + \xi x^2] = \delta F(t)/m$ (Duffing nonlinearity)

$\ddot{x} + \gamma\dot{x} + \Omega_0^2 x[1 + \Omega_0^{-1}\eta x\dot{x} + \epsilon\cos(\Omega_m t) + \xi x^2] = \delta F(t)/m + F_{drive}(t)/m$ (external drive)

$\ddot{x} + \gamma\dot{x} + \Omega_0^2 x[1 + \Omega_0^{-1}\eta x\dot{x} + \epsilon\cos(\Omega_m t) + \xi x^2] + \delta\cos(\omega t)\, y = \delta F(t)/m + F_{drive}(t)/m$ (mode coupling)

$\longrightarrow$ ro-vibrational coupling, non-equilibrium dynamics, cross-Kerr nonlinearities …

$\longrightarrow$ multimode coupling (+rotations), state-dependent noise, chaos, topological physics …

$\longrightarrow$ non-Hermitian dynamics, PT-symmetric processes, …

# SUMMARY

- Active & passive feedback cooling
- Parametric feedback and cold damping
- Quantum control
- Ultrahigh force sensitivity
- Free-fall (sensing of static forces)
- Nonequilibrium dynamics
- Interactions with surfaces (and other particles)
- GHz rotations
- Internal degrees of freedom

# Nanomagnetic Imaging using Scanning Probe Microscopy Techniques

PhD Course, EPFL, 2022

## Sensitivity

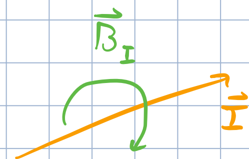
To evaluate sensitivity : Signal - to - noise

Let's take two idealized signals :

- a line of current $\vec{I}$
- a magnetic moment $\vec{m}$

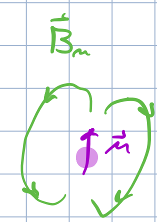
- Current :

$$\vec{B}_I = \frac{\mu_0 \vec{I} \times \vec{r}}{2\pi r^2}$$



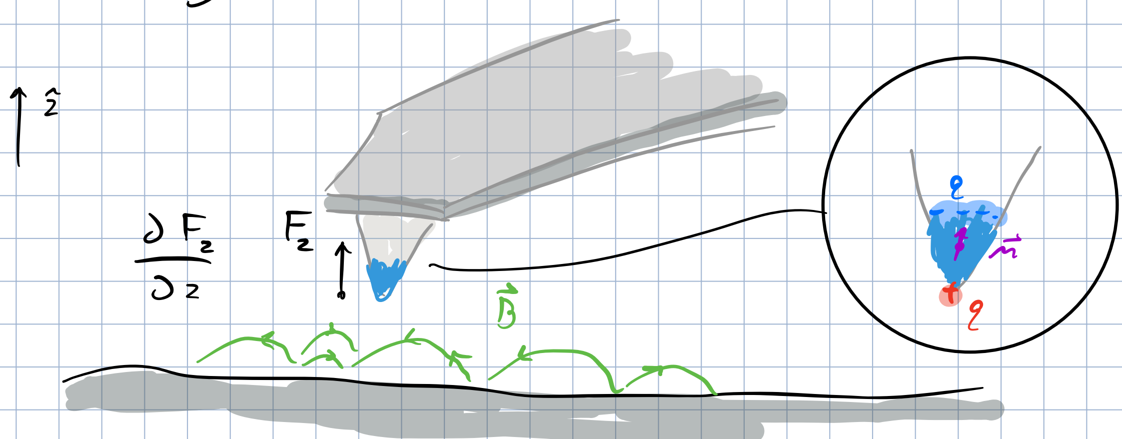

- moment :

$$\vec{B}_m = \frac{\mu_0}{4\pi r^3}\left(\frac{3(\vec{m}\cdot\vec{r})\vec{r}}{r^2} - \vec{m}\right)$$




### MFM

- Signal



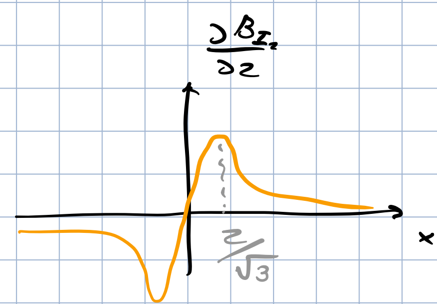
$$F_z = q\vec{B}\cdot\hat{z} + \vec{\nabla}(\vec{\mu}\cdot\vec{B})\cdot\hat{z}$$

($q\vec{B}\cdot\hat{z}$ → monopole; $\vec{\nabla}(\vec{\mu}\cdot\vec{B})\cdot\hat{z}$ → dipole)

If we assume a mono-pole tip and a measurement of force gradients ($\Delta f$):

$$\frac{\partial F_z}{\partial z} = q\frac{\partial B_z}{\partial z} \quad \therefore \quad \frac{\partial B_z}{\partial z} = \frac{1}{q}\frac{\partial F_z}{\partial z}$$

The maximum signal at a given height z that we can measure for a line of current:

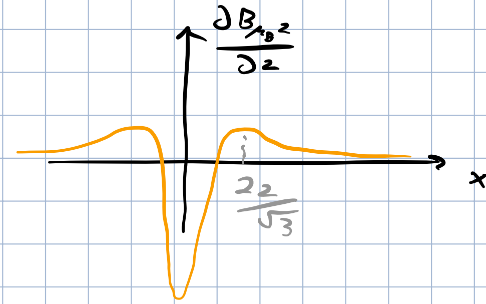



$$\frac{\partial}{\partial x}\left(\frac{\partial B_{I_z}}{\partial z}\right) = 0$$

$$x = -\frac{z}{\sqrt{3}}, \frac{z}{\sqrt{3}}$$

$$\left(\frac{\partial B_{I_z}}{\partial z}\right)_{max} = \frac{3\sqrt{3}\mu_0 I}{16\pi z^2} \qquad [T/m]$$

Similarly for a $\mu_B$ of magnetic moment:




$$\frac{\partial}{\partial x}\left(\frac{\partial B_{\mu_B z}}{\partial z}\right) = 0$$

$$x = -\frac{2z}{\sqrt{3}}, 0, \frac{2z}{\sqrt{3}}$$

$$\left(\frac{\partial B_{\mu_B z}}{\partial z}\right)_{max} = \frac{3\mu_0\mu_B}{2\pi z^4} \qquad [T/m]$$

- Noise

The ultimate noise limit is from the thermal motion of the cantilever:

$$S_F = 4 k_B T \Gamma \quad \leftarrow \text{Fluctuation - Dissipation Theorem}$$

This implies a thermal force noise amplitude that sets a minimum measurable force:

$$F_{min} = \sqrt{4 k_B T \Gamma}$$

For measurements of force gradients done by oscillating the cantilever by $z_{rms}$ and monitoring its resonant frequency, we have:

$$\left(\frac{\partial F}{\partial z}\right)_{min} = \frac{1}{z_{rms}} \sqrt{4 k_B T \Gamma}$$

$$\therefore \left(\frac{\partial B_z}{\partial z}\right)_{min} = \frac{1}{q\, z_{rms}} \sqrt{4 k_B T \Gamma}$$

$30 \frac{T}{m\sqrt{Hz}}$ @ 4K

$\left[\frac{T}{m\sqrt{Hz}}\right]$

We can then see the sensitivity to $I$ or $\mu_B$ by writing:

**Current Sens.**

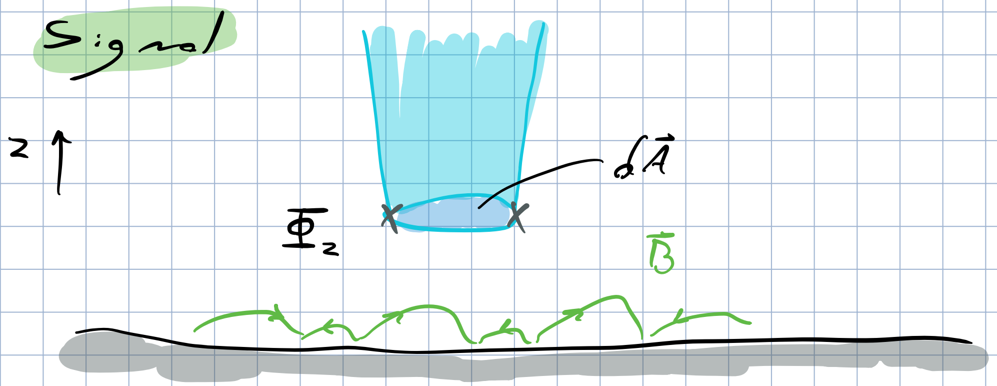
$$\frac{\left(\frac{\partial B_z}{\partial z}\right)_{min}}{\left(\frac{\partial B_{I_z}}{\partial z}\right)_{max}} \cdot I \propto z^2 \quad \left[ A/\sqrt{Hz} \right] \rightarrow \mu A/\sqrt{Hz} \text{ @ } 50\,nm$$

**Moment Sens.**

$$\frac{\left(\frac{\partial B_z}{\partial z}\right)_{min}}{\left(\frac{\partial B_{\mu_B z}}{\partial z}\right)_{max}} \cdot \mu_B \propto z^4 \quad \left[ \mu_B/\sqrt{Hz} \right] \rightarrow 10^3\,\mu_B/\sqrt{Hz} \text{ @ } 50\,nm$$

## SSM

- Signal




$$\Phi_z = \int \vec{B} \cdot d\vec{A}$$

If we now calculate the flux directly above a $\mu_B$ of moment:

$$\left(\Phi_{\mu_B z}\right)_{max} = \frac{\mu_0 \mu_B R^2}{2(z^2 + R^2)^{3/2}}$$

$R$ → loop radius

$[T \cdot m^2 = Wb]$

- **Noise**

There are several sources of noise:

- Johnson noise
- shot noise
- 1/f noise ← at low freq
- quantum noise → $\Phi_Q = \sqrt{\hbar L}$ ($L$ ← Loop inductance)

state of the art are ~ 4× this limit

$$\therefore \left(\Phi_z\right)_{min} = \Phi_{noise}$$

→ $50 \sim \Phi_0 / \sqrt{Hz}$

Sensitivity:

Current Sens.

$$\frac{\left(\Phi_z\right)_{min}}{\left(\Phi_{Iz}\right)_{max}} \cdot I \quad \left[A/\sqrt{Hz}\right]$$

→ $10\ nA/\sqrt{Hz}$ @ 50 nm

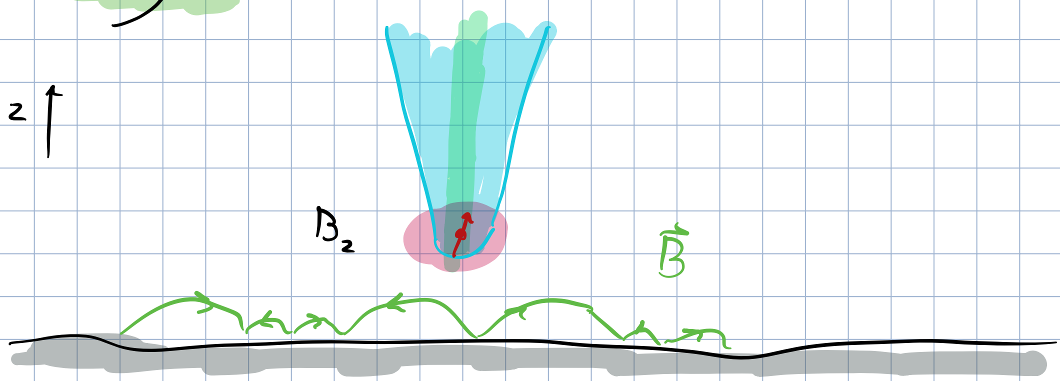
Moment Sens.

$$\frac{\left(\Phi_z\right)_{min}}{\left(\Phi_{\mu z_B}\right)_{max}} \cdot \mu_B \propto z^3 \quad \left[\mu_B/\sqrt{Hz}\right]$$

→ $\mu_B/\sqrt{Hz}$ @ 50 nm

# SNVM

- Signal




By measuring the NV splitting, we measure the magnetic field along the NV axis:

$$B_z = \vec{B} \cdot \hat{z}$$

$$\left(B_{I z}\right)_{max} = \frac{\mu_0 I}{4\pi z} \quad [T]$$

$$\left(B_{\mu_B z}\right)_{max} = \frac{\mu_0 \mu_B}{2\pi z^3} \quad [T]$$

- **Noise**

SNVM is typically limited by photon shot noise from the optical read-out. Minimum measurable field can be written as:

$$(B_z)_{min} = \frac{1}{\gamma \, \epsilon \sqrt{I_0 \, t_{acq} \, T_2}}$$

(optical contrast → $\epsilon$; gyromagnetic ratio → $\gamma$; count rate → $I_0$; integration → $t_{acq}$; decoherence → $T_2$) → $100 \, nT/\sqrt{Hz}$

Sensitivity:

Current Sens. $\dfrac{(B_z)_{min}}{(B_{I_z})_{max}} \cdot I \propto z^2$ $\left[ A/\sqrt{Hz} \right]$ → $10 \, nA/\sqrt{Hz}$ @ 25 nm

Moment Sens. $\dfrac{(B_z)_{min}}{(B_{\mu_0 z})_{max}} \cdot \mu_B \propto z^3$ $\left[ \mu_B/\sqrt{Hz} \right]$ → $\mu_B/\sqrt{Hz}$ @ 25 nm

# Reconstruction of $\vec{I}$ + $\vec{m}$ from $\vec{B}$

Biot - Savart :

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} d^3 \vec{r}\,'$$

For current density $\vec{J} = J_x \hat{x} + J_y \hat{y}$

or magnetization $\vec{m} = M_z \hat{z}$ in 2D

we can write this in k-space :

$$\tilde{B}_z(k_x, k_y, z) = \underbrace{i \frac{1}{2} \mu_0 d e^{-kz}}_{g(k,z)} \left[ \frac{k_y}{k} \tilde{J}_x(k_x, k_y) - \frac{k_x}{k} \tilde{J}_y(k_x, k_y) \right]$$

w/ $k = \sqrt{k_x^2 + k_y^2}$

and $d \ll z$

↑ film thickness

Continuity equation : $\vec{\nabla} \cdot \vec{J} = 0$

$$\longrightarrow \quad k_x \tilde{J}_x + k_y \tilde{J}_y = 0$$

$$\tilde{J}_y = - \frac{k_x}{k_y} \tilde{J}_x$$

Together :

$$\tilde{B}_z = i g \frac{\tilde{J}_x}{k} \left( k_y + \frac{k_x^2}{k_y} \right) = i g \frac{\tilde{J}_x}{k_y} k$$

$$\tilde{J}_x = -\frac{i k_y \tilde{B}_z}{k g}$$

$$\tilde{J}_y = \frac{i k_x \tilde{B}_z}{k g}$$

Magnetization :

$$\vec{J} = \vec{\nabla} \times \vec{m}$$

$$\therefore \quad J_x = \frac{\partial m_z}{\partial y} , \quad J_y = -\frac{\partial m_z}{\partial x}$$

$$\tilde{J}_x = -i k_y \tilde{m}_z , \quad \tilde{J}_y = i k_x \tilde{m}_z$$

$$\therefore \quad \tilde{m}_z = \frac{\tilde{B}_z}{k g}$$

**Current density**

**Magnetization**

$$\tilde{B}_z = -i \frac{1}{2} \mu_0 d \, e^{-kz} \frac{k}{k_x} \tilde{J}_y \qquad \tilde{B}_z = \frac{1}{2} \mu_0 d \, e^{-kz} k \, \tilde{m}_z$$

Derivative along z :

$$\frac{\partial \tilde{B}_z}{\partial z} \propto e^{-kz} \frac{k^2}{k_x} \qquad \frac{\partial \tilde{B}_z}{\partial z} \propto e^{-kz} k^2$$

$$k \propto k_x \propto \frac{1}{\lambda}$$

→ Feature size

Normalized to distance : $\frac{\lambda}{z}$

$$\tilde{B}_z \propto e^{-\frac{z}{\lambda}}$$

$$\tilde{B}_z \propto \frac{1}{z}\left(\frac{z}{\lambda}\right) e^{-\frac{z}{\lambda}}$$

$$\frac{\partial \tilde{B}_z}{\partial z} \propto \frac{1}{z}\left(\frac{z}{\lambda}\right) e^{-\frac{z}{\lambda}}$$

$$\frac{\partial \tilde{B}_z}{\partial z} \propto \frac{1}{z^2}\left(\frac{z}{\lambda}\right)^2 e^{-\frac{z}{\lambda}}$$