Lecture 14 (16.12.2020)

Exam

- Oral (to be scheduled)
- On all the main topics (review to follow)
- Start w/ presentation of a recent paper (to be chosen from a short list) ... discussion will start from there. ← Example of what we want.

Review

- General principles of stress - strain
- Force & Torque Balance
- Application of boundary conditions
- Flexural vibrations (of a cantilever)
- Zener's model for an anelastic solid
- Correspondence btw. beam dynamics and harmonic oscillator

Beam

Force at end

motion at end

$$\hat{x}_{end}(\omega) = \frac{4\hat{F}_p(\omega)}{m} \frac{1}{\omega_1^2 - \omega^2 + i\frac{\omega_1^2}{Q}}$$

$$\hat{x}_{end}(\omega) = \hat{F}_p(\omega)\, \chi_m(\omega)$$

Harm. Osc.

$$\hat{x}(\omega) = \frac{\hat{F}(\omega)}{m} \frac{1}{\omega_0^2 - \omega^2 + i\frac{\omega_0 \omega}{Q}}$$

$$\hat{x}(\omega) = \hat{F}(\omega) \frac{\chi}{m}(\omega)$$

- Static & Dynamic spring constant:

$$k_D \approx \frac{i}{Q} k_s$$

- Concept of power spectral density (PSD):

$$S_x(\omega) = \int_{-\infty}^{\infty} \langle x(t)\, x(0) \rangle \, e^{i\omega t} \, dt$$

Power Spectral Density

Correlation Function

$$\langle x^2 \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_x(\omega) \, d\omega$$

$$S_x(\omega) = \frac{S_F(\omega)}{m^2} \left(\frac{1}{(\omega_0^2 - \omega^2)^2 + \frac{\omega_0^2 \omega^2}{Q^2}} \right)$$

- Fluct.-disp. theorem:

$$\bar{S}_f(\omega) = 4 k_B T \Gamma$$

$$\hookrightarrow \quad F_{min} = \sqrt{4 k_B T \Gamma} \quad \left[\frac{N}{\sqrt{Hz}}\right]$$

$$m_{min} = \frac{2}{x_0 \omega_0^2} \sqrt{2 k_B T \Gamma} \quad \left[\frac{kg}{\sqrt{Hz}}\right]$$

From : $\frac{\delta \nu_0}{\nu} = -\frac{1}{2}\frac{\delta m}{m}$

$$k_{min} = \frac{2}{x_0}\sqrt{2 k_B T \Gamma} \quad \left[\frac{N}{m\sqrt{Hz}}\right]$$

From : $\frac{\delta \nu_0}{\nu} = \frac{1}{2}\frac{\delta k}{k}$

- Transducers (typically nanomechanical elements)
  Detectors (many different kinds for displacement)

- Motivations for ground-state cooling ($\hbar\omega_0 \gg k_B T$)
  - ultimate force resolution
  - quantum regime for "macroscopic" objects
  - Measure mechanical superpositions and coherences

  → Methods :
  - "Brute force"
  - Damping
  - Cavity cooling

- Standard Quantum Limit :

$$\Delta x_{SQL} = x_{ZPF} = \sqrt{\langle x^2 \rangle_0} = \sqrt{\frac{\hbar}{2m\omega_0}}$$

- How to achieve cooling by damping?

reduce disp.

$$N_{mode,min} = \frac{1}{\hbar}\sqrt{\Gamma k_B T \bar{S}_{xx}} = \frac{1}{2\hbar}\sqrt{\bar{S}_F \bar{S}_{xx}}$$

reduce starting temp.

reduce detector noise

- Cavity cooling ≠ Quantum treatment of Fluct. - Disp. $\longrightarrow$ not so emphasized

Quantum PSD:

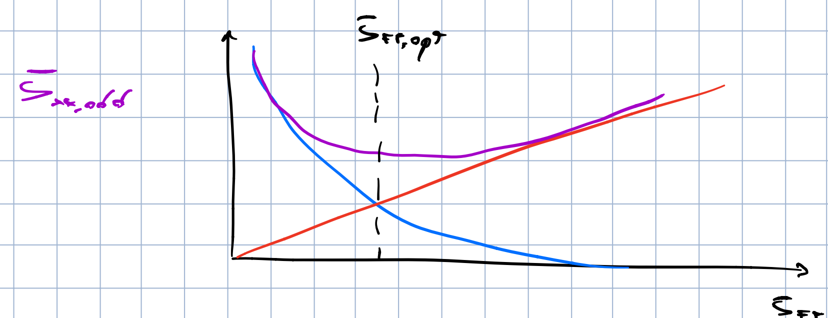
$$S_{xx}(\omega) = \int_{-\infty}^{\infty} \langle \hat{x}(t)\,\hat{x}(0) \rangle \, e^{i\omega t}\, dt$$

- Quantum limit on the noise of a displacement detector:

$$\frac{\hbar}{2} = \sqrt{S_{FF}\, S_{xx}^{I}}$$

- Total measurement noise:

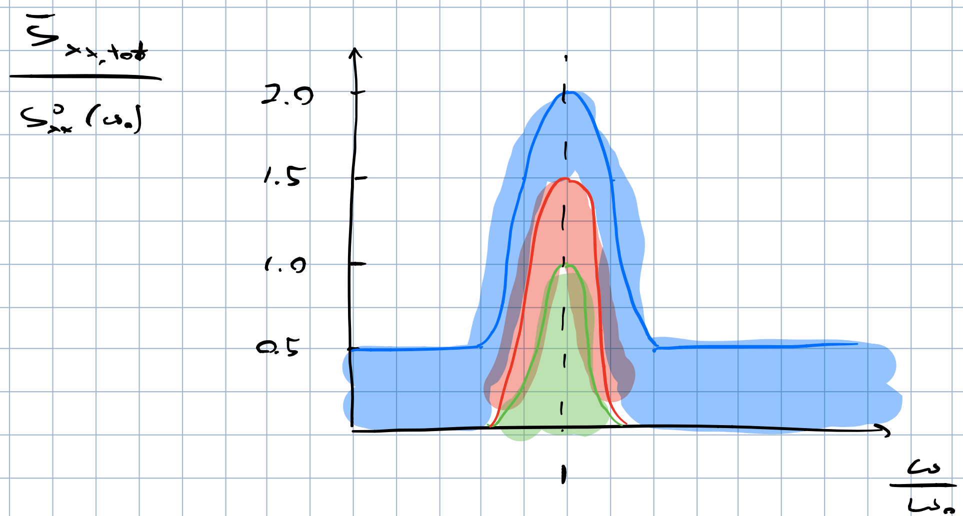
$$\bar{S}_{xx,tot}(\omega) = \underbrace{\bar{S}_{xx}^{0}(\omega)}_{\text{zero-point motion}} + \overbrace{\underbrace{S_{xx}^{I}(\omega)}_{\text{measurement imprecision}} + \underbrace{|\chi_m(\omega)|^2\, \bar{S}_{FF}(\omega)}_{\text{back-action}}}^{\bar{S}_{xx,add}}$$



On resonance :

$$\bar{S}_{xx,tot}(\omega_0) = \left[1 + \frac{1}{2} + \frac{1}{2} \right] S^{0}_{xx}(\omega_0)$$

$$= 2\, S^{0}_{xx}(\omega_0) = \frac{4t}{\omega_0 \Gamma}$$



- Mechanical dissipation :

$$\frac{dE}{dt} = -\frac{\Gamma}{m} E$$

$$\Gamma = \frac{m\omega_0}{Q}$$

Def. of Q $\longrightarrow$ $Q = 2\pi \dfrac{\text{Energy stored}}{\text{Energy disp. in one period}}$

- Concept of damping dilution:

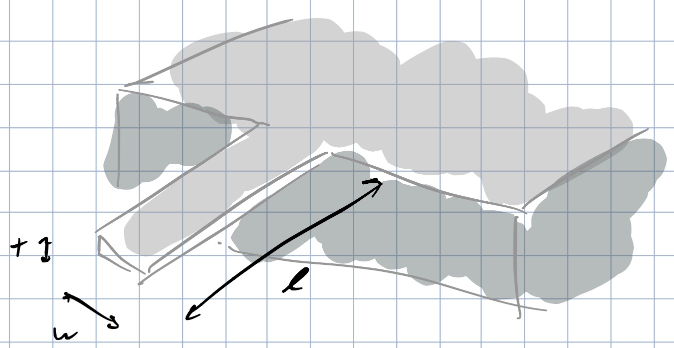
  increase energy stored; keep energy loss per cycle the same

- Why now?

  For small F_{min}, k_{min}, we need small Γ.

$$\Gamma = \frac{m\omega_0}{Q}$$

For beam: $\omega_0 \propto \frac{t}{l^2}$



$Q \propto t$ ← exp. observed

$m \propto wtl$

$$\therefore \Gamma \propto \frac{wt\cancel{l}\cdot\frac{\cancel{t}}{l^{\cancel{2}}}}{\cancel{t}}$$

$$\Gamma \propto \underbrace{\frac{wt}{l}}_{\text{long and thin and small}}$$

If we scale uniformly each dimension by β:

$\Gamma \propto \beta$ → Smaller, less energy loss