Introduction to Nanomechanics Spring Semester 2012
Exercise Sheet 5 submission due on Monday, 8.5.2012, 12:00 am

1 Main Properties of a Real Cantilever

The cantilevers used for our experiments in the lab are fabricated out of single-crystal silicon (Si) and have a thickness of 100 nm, width 3 μm and length 120 μm. The Q-factor is measured to be 50 000. The density ρ and Young's modulus E for Si are 2330 kg/m^3 and $1.69 \cdot 10^{11}$ N/m^2 respectively. Assume that the cantilever is at room temperature and the measurement bandwidth is 1 Hz.

1. Calculate the gravity force acting on the cantilever. (1 point)

2. Calculate the fundamental eigenfrequency ν_0 and the natural linewidth of the cantilever resonance. (2.5 points)
   (The natural linewidth is the FWHM of the spectral density of the cantilever, which is, in general, different from the spectral density of a simple harmonic oscillator.)

3. Calculate Δx_{SQL} and the *rms* displacement fluctuations for the cantilever. (2 points)

4. Calculate the force sensitivity and the minimum detectable force of the cantilever limited by thermal fluctuations. (1.5 points)

5. Consider now the situation in which a cubical shaped SmCo magnetic tip is glued to the end of the cantilever, and interacts with an electron spin of a sample located 100 nm away from the cantilever end in the same direction of its length (assume this to be the $\hat{\boldsymbol{x}}$ direction).

   Assume the magnetic tip being a cube of 1 μm side and having a magnetization density $\mu = 10^6$ A/m along the $\hat{\boldsymbol{x}}$ direction; assume the electron spin having a magnetic moment $\mu_e = 9.3 \cdot 10^{-24}$ JT^{-1} along the $\hat{\boldsymbol{x}}$ direction. Treating both the tip and the spin as classical magnetic dipoles, estimate the force acting on the cantilever. (2 points)

6. The magnetic moment due to a particle spin, $\boldsymbol{\mu}$, can be in general obtained from:
   $$\boldsymbol{\mu} = g \frac{e}{2m} \boldsymbol{S} \qquad (1)$$
   where the g-factor is ≈ -2 for an electron and ≈ 5.586 for a proton, e is the elementary charge, $S = \frac{1}{2}\hbar$ is the value of the spin for both electrons and protons, and m is their rest mass, which for a proton is 1836 times bigger than that of an electron.

   Given this, calculate the ratio between the magnetic moments of an electron and of a proton spin. (1 point)