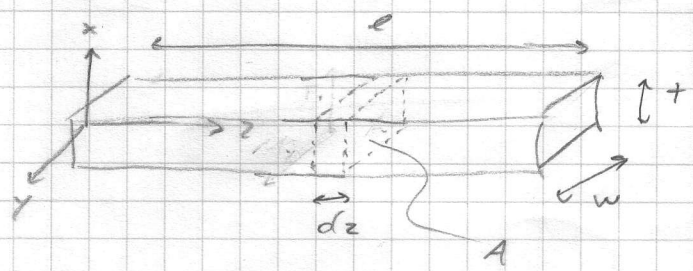


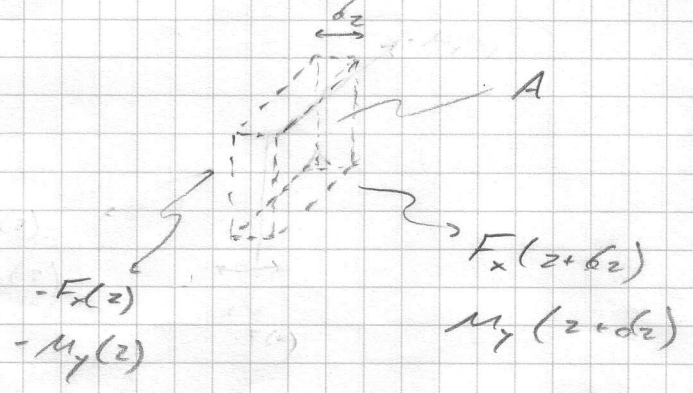
Flexural Vibrations



Let us calculate the dynamic behavior of this beam as the end forces in the x-direction

Balance Forces

$$F_x(z+dz) - F_x(z) = \underbrace{\rho A dz}_{\text{mass}} \underbrace{\frac{d^2 u_x}{dt^2}}_{\text{acc.}}$$



Balance Torque

$$M_y(z+dz) - M_y(z) + F_x(z+dz) dz = 0$$

Expanding out for small dz around z:

$$\left[\begin{aligned} \frac{\partial F_x}{\partial z} &= \rho A \frac{\partial^2 u_x}{\partial t^2} \\ \frac{\partial M_y}{\partial z} &= -F_x(z) \end{aligned} \right]$$

Recall from lecture #1: $\frac{d^2 u_x}{dz^2} = \frac{M_y}{EI_y}$

$$M_y = EI_y \frac{\partial^2 u_x}{\partial z^2}$$

$$\frac{\partial^2}{\partial z^2} \left(EI_y \frac{\partial^2 u_x}{\partial z^2} \right) = -\rho A \frac{\partial^2 u_x}{\partial t^2}$$

$$\left[EI_y \frac{\partial^4 u_x}{\partial z^4} + \rho A \frac{\partial^2 u_x}{\partial t^2} = 0 \right]$$

