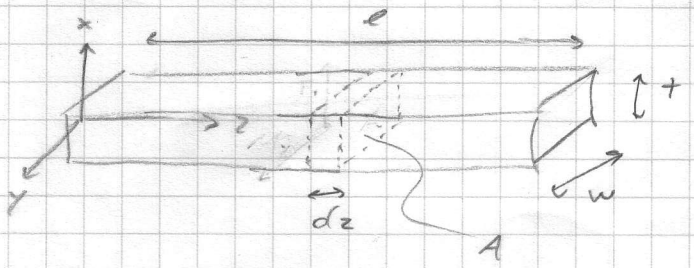
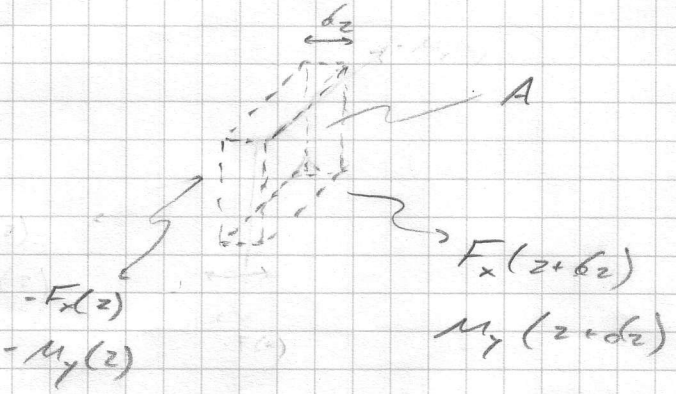
Flexural Vibrations



Let us calculate the dynamic behavior of this beam as the end forces in the x-direction



Balance forces

$$F_x(z+dz) - F_x(z) = \rho A dz \frac{d^2 u_x}{dt^2}$$

mass acc.

Balance Torque

$$M_y(z+dz) - M_y(z) + F_x(z+dz)\,dz = 0$$

Expanding out for small dz around z:

$$\frac{\partial F_x}{\partial z} = \rho A \frac{\partial^2 u_x}{\partial t^2}$$

$$\frac{\partial M_y}{\partial z} = -F_x(z)$$

Recall from lecture #1: $\frac{d^2 u_x}{dz^2} = \frac{M_y}{E I_y}$

$$M_y = E I_y \frac{\partial^2 u_x}{\partial z^2}$$

$$\frac{\partial^2}{\partial z^2}\left(E I_y \frac{\partial^2 u_x}{\partial z^2} \right) = -\rho A \frac{\partial^2 u_x}{\partial t^2}$$

$$E I_y \frac{\partial^4 u_x}{\partial z^4} + \rho A \frac{\partial^2 u_x}{\partial t^2} = 0 \quad \longleftarrow$$