Fluids

Hydrostatics

$$P = \frac{F}{A} \qquad \left[\frac{N}{m^2} = Pa \right]$$

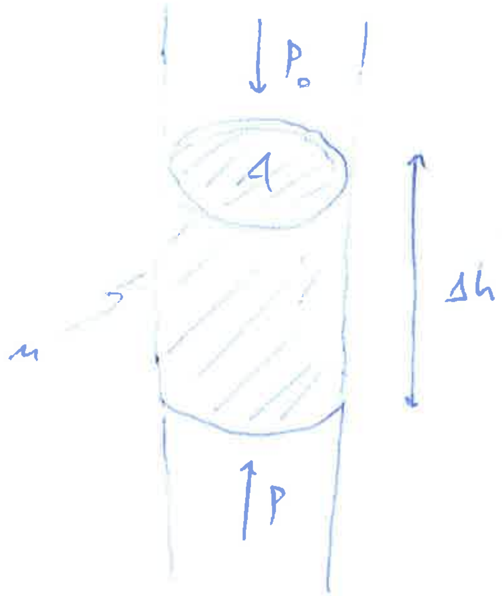
pressure (P), force (F), area (A)

Exp. shot on cup of water

Exp. manometer all around enclosure

Fluids and solids are hard to compress; gases are easy. This means that K (bulk modulus) is large and κ (compressibility) is small for solids and fluids.

Recall: $$\frac{\Delta V}{V} = -\kappa \cdot \Delta p$$



Therefore, in equilibrium:

$$PA = P_0 A + mg$$

$$m = \rho A \Delta h$$

ρ → density

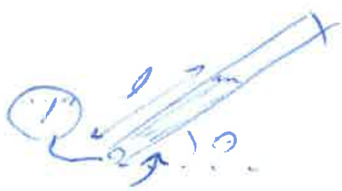
$$PA = P_0 A + \rho A \Delta h g$$

$$P = P_0 + \rho g \Delta h$$

Exp

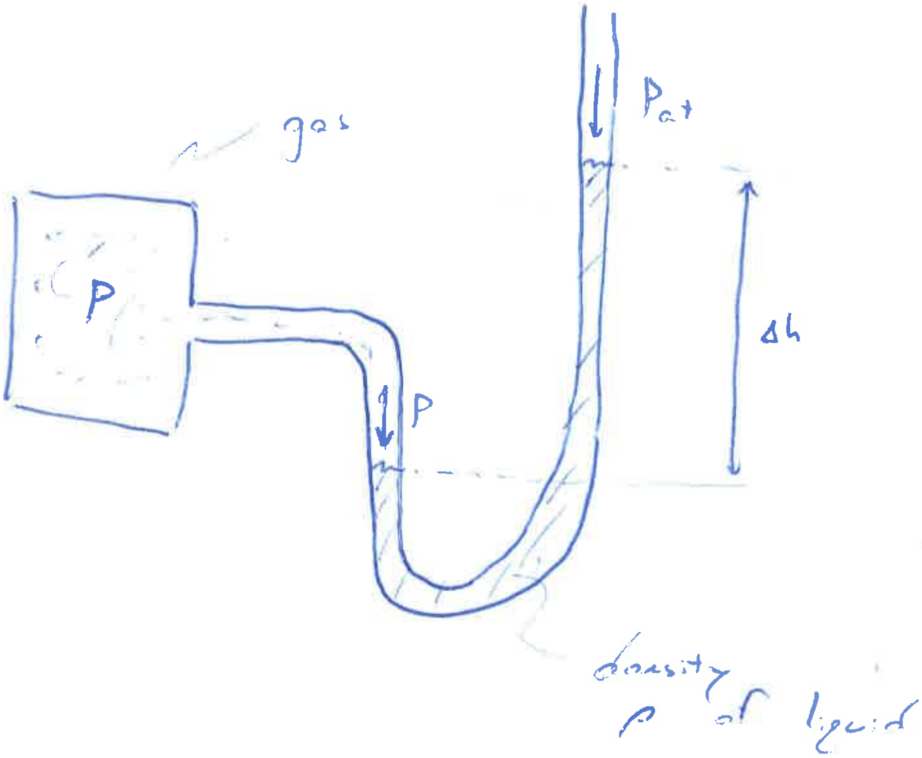
Pressure of water column

Exp



Pressure of rotatable water column: depends only on height of level, i.e. $l \sin\theta = h$

Manometer



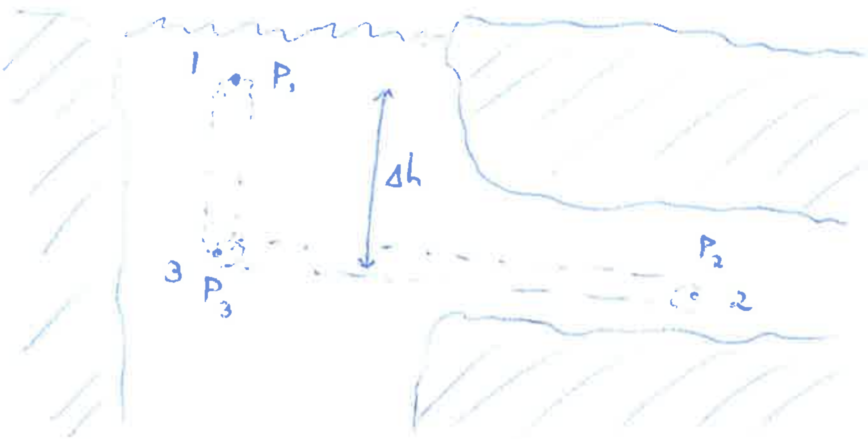
Equilibrium :

$$PA = P_{at}A + mg$$

$$PA = P_{at}A + \rho \Delta h A g$$

$$P = P_{at} + \rho g \Delta h$$

measure of P



$$P_3 A = P_1 A + \rho g \Delta h A$$

$$P_3 = P_1 + \rho g \Delta h$$

$$P_3 A = P_2 A$$

$$P_3 = P_2 = P_1 + \rho g \Delta h$$

→ Gravity Pressure (Schweredruck) in liquid:

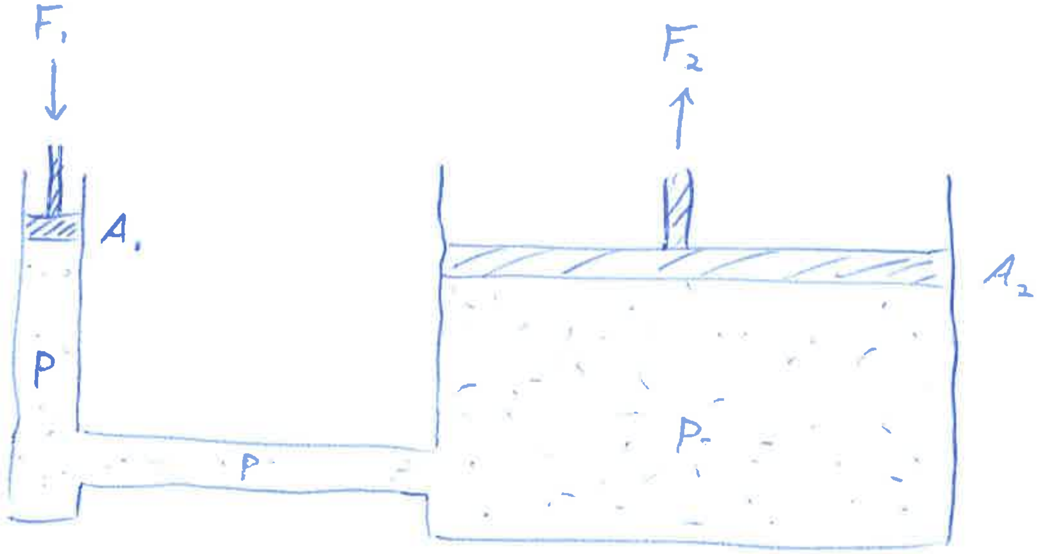
$$P = \rho g \Delta h$$

ρ: density of liquid

Pascal's Principle

A pressure change applied to a confined fluid is transmitted undiminished to every point in the fluid and to the walls of the container.

Hydraulic Lift



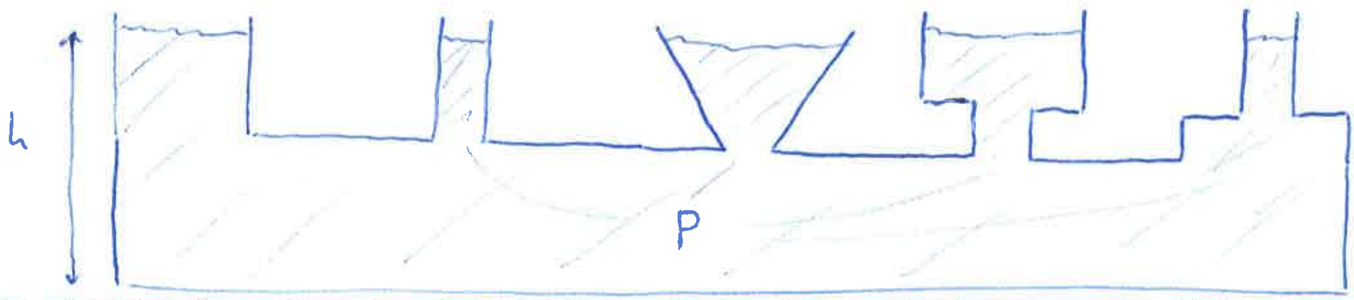
$$F_1 = P A_1 \longrightarrow P = \frac{F_1}{A_1}$$

$$F_2 = P A_2 \longrightarrow F_2 = \frac{F_1}{A_1} A_2$$

$$\therefore \quad \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Exp Dentist chair

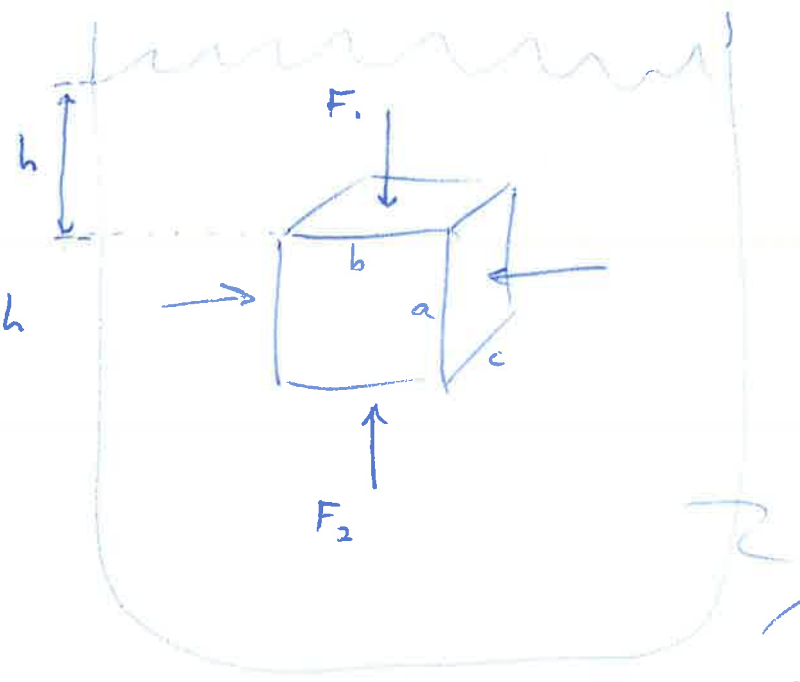
Hydrostatic Paradox



Buoyant Force (Auftrieb)

Archimedes' Principle

A body submerged in a fluid is buoyed up by a force equal to the weight of the displaced fluid.



$$F_A = F_2 - F_1$$

$$F_A = bc\rho g(h+a) - bc\rho g h$$

$$F_A = bc\rho g a$$

$$\boxed{F_A = \rho g V}$$

$V = abc$

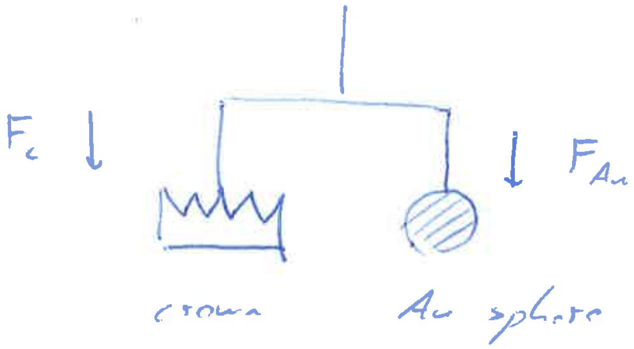
ρ : density of liquid

Exp

Buoyancy in air w/ glass flask w/ and w/o air.

Show Archimedes' crown slide. Here is how Archimedes decided to test whether the crown was Au ~~?~~ or something cheaper (i.e. less dense):

①



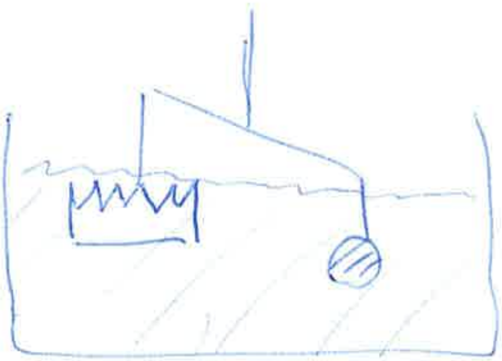
$$F_c = F_{Au}$$

$$m_c g = m_{Au} g$$

$$\rho_c V_c g = \rho_{Au} V_{Au} g$$

$$\rho_c = \frac{\rho_{Au} V_{Au}}{V_c}$$

②



Buoyancy Forces:

$$F_{A,c} = \rho g V_c$$

$$F_{A,Au} = \rho g V_{Au}$$

ρ: density of water

(——>)

(9)

If $F_{A,c} > F_{A,Au}$, then

$$\rho g V_c > \rho g V_{Au}$$

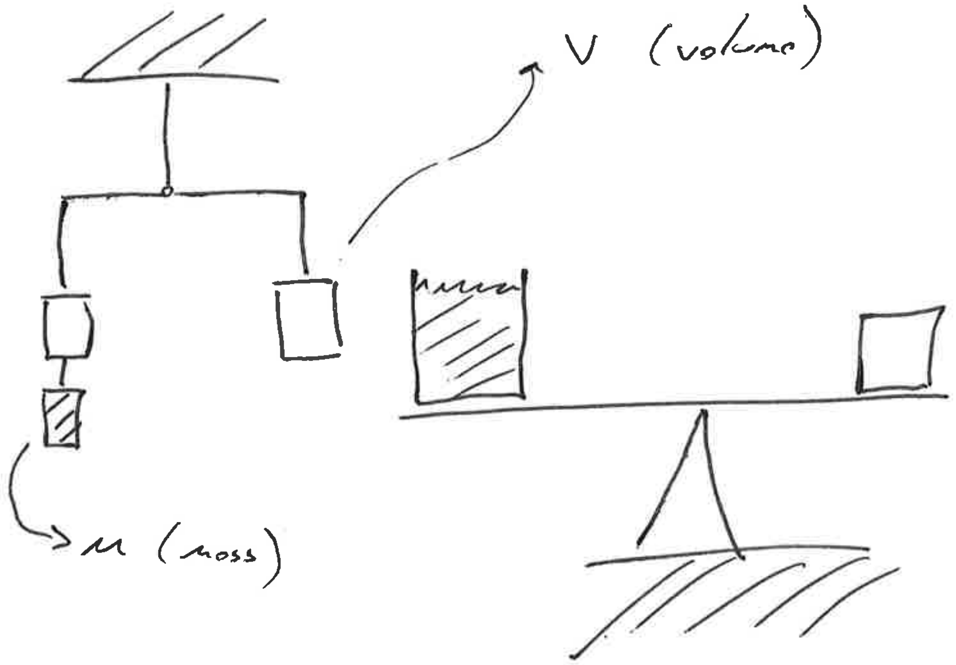
$$V_c > V_{Au}$$

$$\therefore \rho_c < \rho_{Au}$$

The crown is less dense than Au. It isn't made of pure Au!

Exp

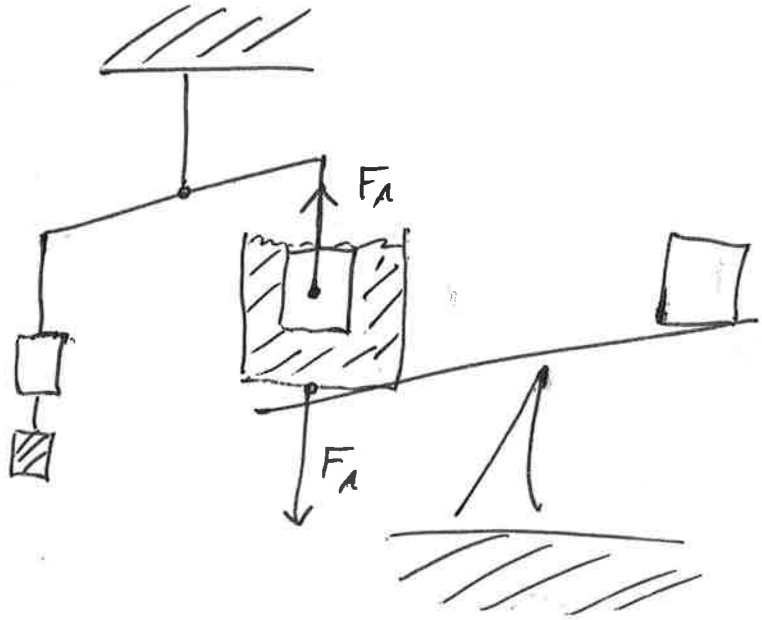
①



Equilibrium

$\sum F = 0$

②

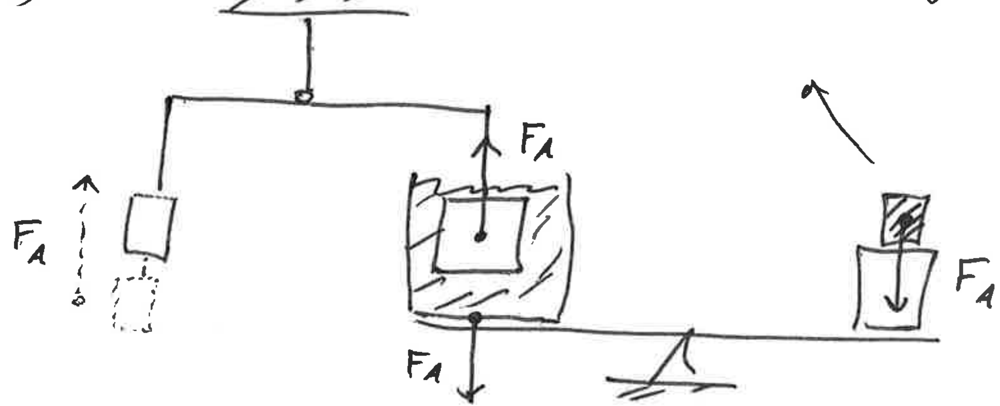


Out of Equilibrium

$\sum F \neq 0$

③

$F_A = mg = \rho_w V g$



Equilibrium

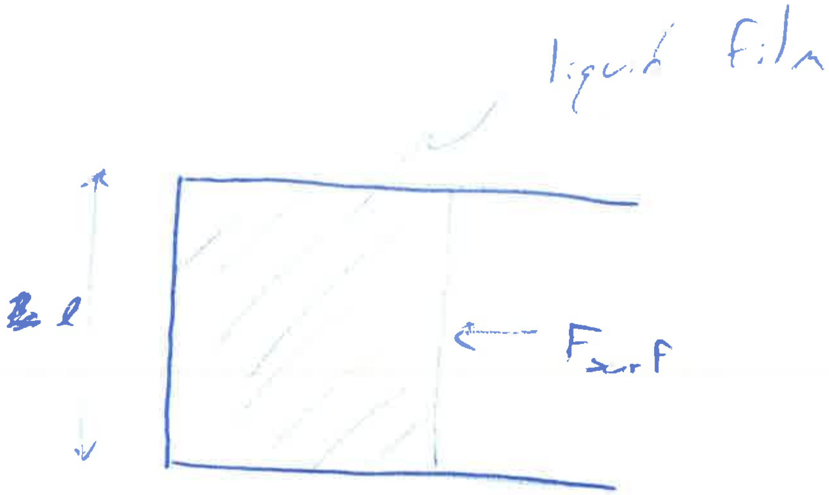
$\sum F = 0$

Surface Tension (Oberflächenspannung)

$$\sum_i F_i = F_{surf} \neq 0$$

$$\sum_i F_i = 0$$

Arises b/c of cohesive forces in liquid.



$$F_{surf} = \sigma \, l$$

σ → surface tension

$$\sigma = \frac{F_{surf}}{l}$$

this is a constant related to the liquid (material).

(show slide/video)

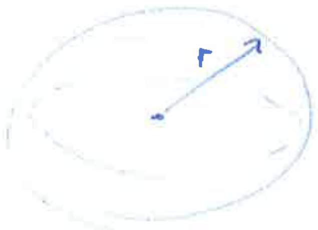
Exp

① Surf. tension of film

② Surf. tension w/ circle

③ Surf. tension of liquid droplet

Droplet of Liquid



Consider one half of the droplet:

In equilibrium:

$$F_{surf} = F_p$$

$$\sigma \cdot 2\pi r = p \, \pi r^2 \rightarrow p = \frac{2\sigma}{r}$$

Dropper

In equilibrium:

$$F_g = F_{surf}$$

$$mg = 2\pi r \sigma$$

$$\rho V g = 2\pi r \sigma$$

$$\therefore \boxed{V = \frac{2\pi r \sigma}{\rho g}}$$

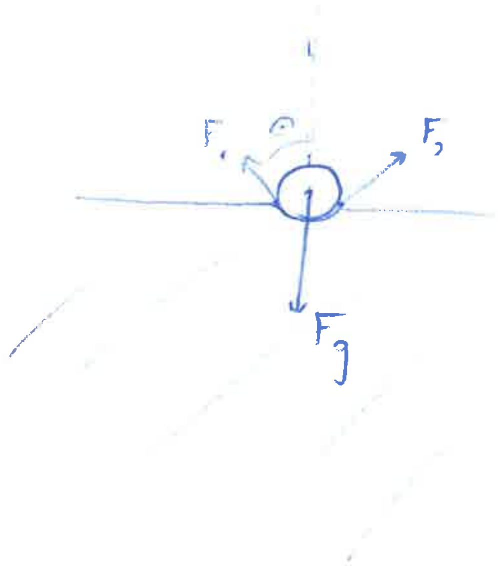
→ Each drop of liquid has

Exp

20 drops ethanol
20 drops water } unequal volume

Example of Surface Tension

Floating needle



$$F_g = F_1 + F_2$$

$$mg = l\sigma \cos\theta + l\sigma \cos\theta$$

$$mg = 2l\sigma \cos\theta$$

$$\cos\theta = \frac{mg}{2l\sigma}$$

$$\theta = \cos^{-1}\left(\frac{mg}{2l\sigma}\right)$$

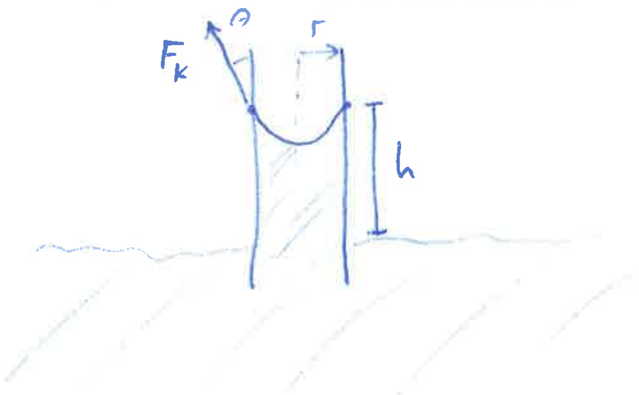
$\therefore$ For the need to float, $mg \leq 2l\sigma$

$$\boxed{m \leq \frac{2l\sigma}{g}}$$

Capillary Action (Kapillarität)

Arises b/c of adhesive forces between liquid molecules and solid molecules.

adhesive forces > cohesive forces



In equilibrium

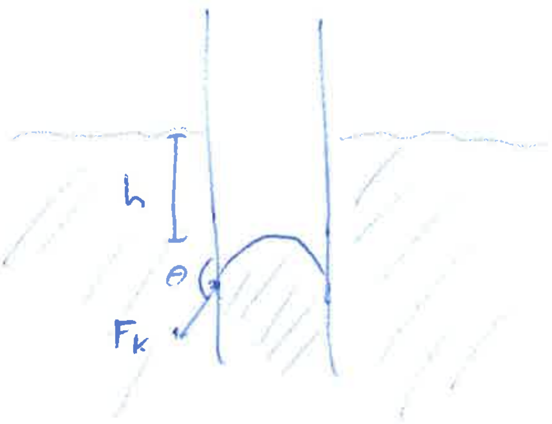
$$F_g = F_k \cos\theta$$

$$\pi r^2 h \rho g = 2\pi r \sigma \cos\theta$$

$$h = \frac{2\sigma\cos\theta}{r\rho g}$$

$0 < \theta < \frac{\pi}{2}$

liquid attracted more to glass than themselves.



cohesive forces > adhesive forces

$\frac{\pi}{2} < \theta < \pi$

liquid attracted more to themselves than glass.