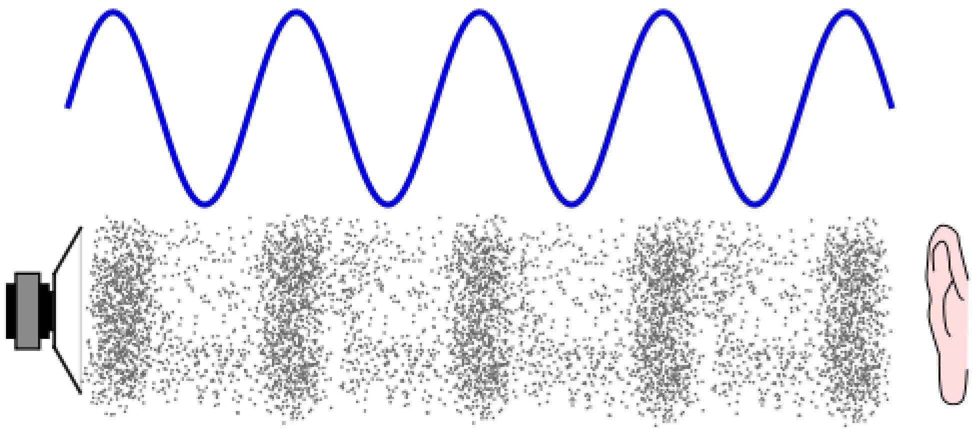
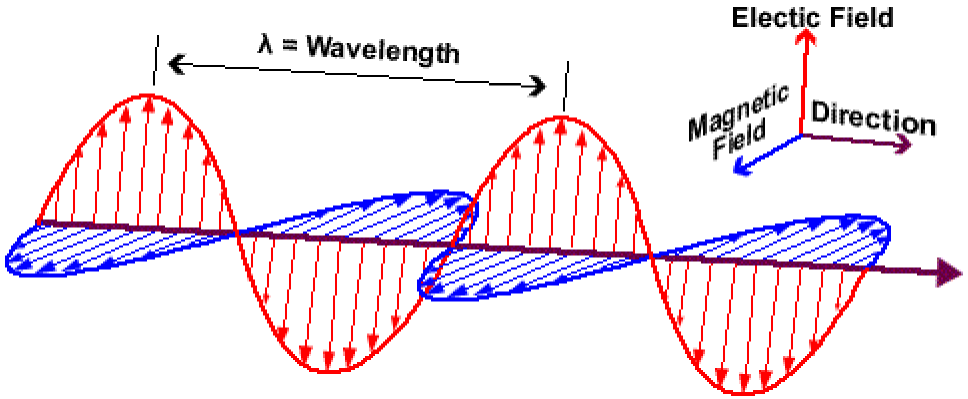
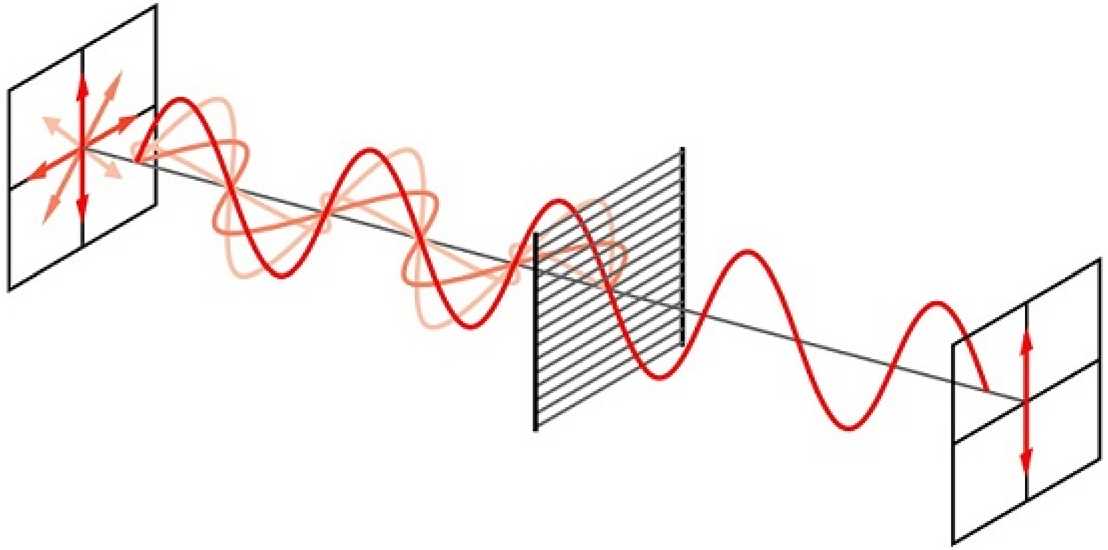
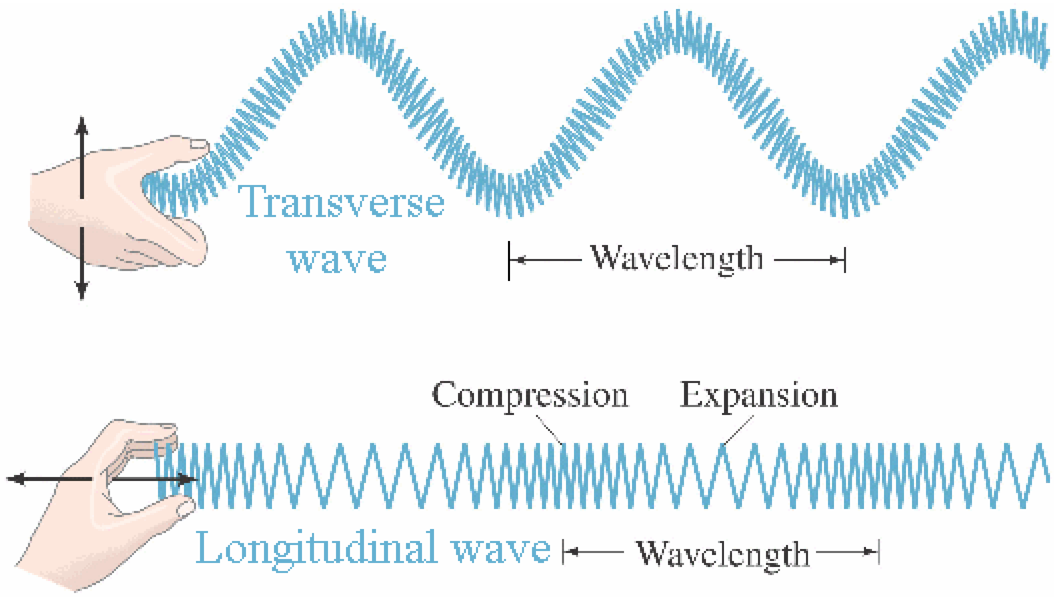
Introduction to Physics I

For Biologists, Geoscientists, & Pharmaceutical Scientists




λ = Wavelength
Electic Field
Magnetic Field
Direction




Transverse
wave
Wavelength
Compression
Expansion
Longitudinal wave
Wavelength

[illegible]

[illegible]

[illegible]

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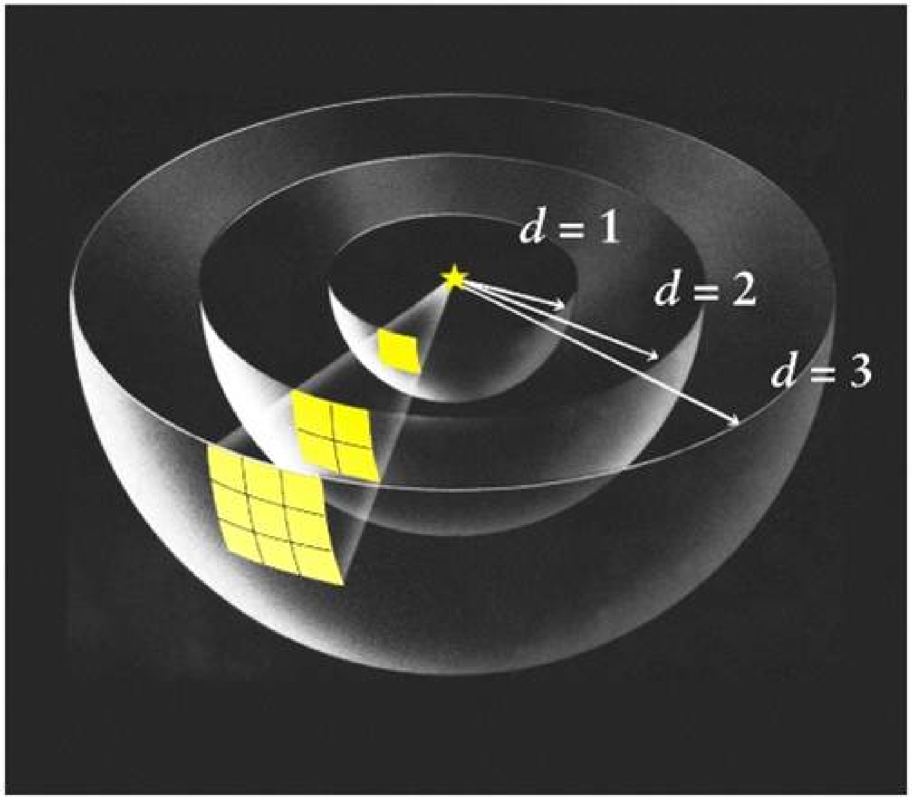


d = 1
d = 2
d = 3

Waves I

- Last few lectures, we studied oscillation.
- Now we study travelling oscillations, i.e. waves.
- Waves travel through various media (water, air, land, space) transporting energy and momentum, but no mass.
- Described by a function of position and time:

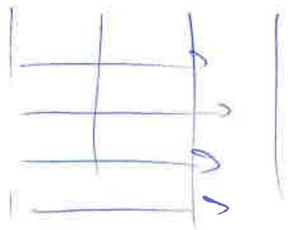
$$A = A(x, y, z, t)$$

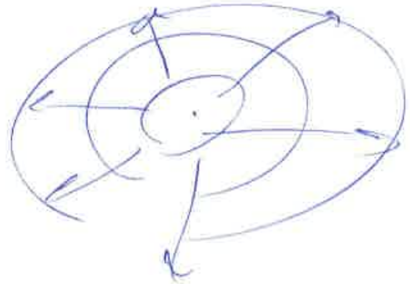
- Waves are propagated by the coupling between local oscillations in a medium.

| Exp: Coupled Pendula |

- Examples: Ocean waves, sound waves, electromagnetic waves, rope waves.
- Wave oscillations can be ~~sc~~ scalar or vectorial.
  - vectorial → light waves
  - scalar → sound waves
- Transverse and Longitudinal Waves
  - Transverse → oscillation transverse to propagation
  - Longitudinal → oscillation along propagation

• Plane waves / Circular waves





• Polarized vs. Unpolarized waves

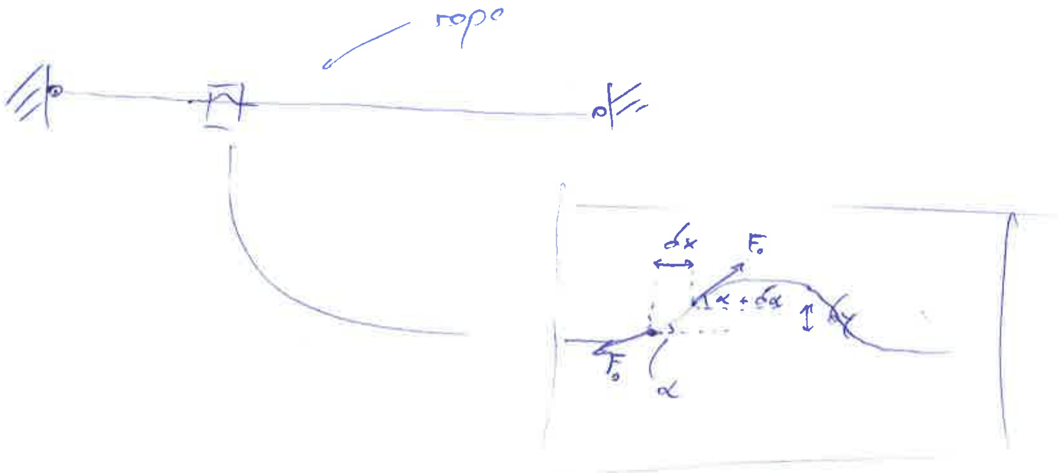
↳ e.g. light

Pol.

Unpol.

Exp: Wave-pool
Exp: Torsion wave

Mathematical Description of Waves



Exp: Rope Wave

$$F_y = F_0 \sin(\alpha + \delta\alpha) - F_0 \sin\alpha$$

For small α:

$$F_y = F_0(\alpha + \delta\alpha) - F_0\alpha = F_0\,\delta\alpha$$

$$\alpha = \frac{\delta y}{\delta x}$$

$$\frac{\delta\alpha}{\delta x} = \frac{\delta^2 y}{\delta x^2}$$

$$\therefore\ F_y = F_0 \frac{\delta^2 y}{\delta x^2}\,\delta x$$

Also:

$$F_y = \delta m \cdot a_y = \delta m \frac{\delta^2 y}{\delta t^2}$$

$$\rightarrow \delta m = \rho A\,\delta x$$

(ρ: density; A: x-sectional area)

$$\therefore\ \rho A\,\delta x \frac{\partial^2 y}{\partial t^2} = F_0 \frac{\partial^2 y}{\partial x^2}\,\delta x$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{F_0}{\rho A} \frac{\partial^2 y}{\partial x^2}$$

$$y = f(\underbrace{v \cdot t - x}_{u}) \qquad \frac{\partial y}{\partial t} = \frac{\partial y}{\partial u} \cdot \frac{\partial u}{\partial t}$$

$$\frac{\partial y}{\partial t} = f'(v \cdot t - x) \cdot v$$

$$\frac{\partial^2 y}{\partial t^2} = v \frac{\partial}{\partial t}\left(f'(v \cdot t - x)\right) = v \cdot f''(v \cdot t - x) \cdot v$$

$$\frac{\partial^2 y}{\partial t^2} = v^2 f''(v \cdot t - x) \quad \longleftarrow$$

$$\frac{\partial y}{\partial x} = f'(v \cdot t - x) \cdot (-1)$$

$$\frac{\partial^2 y}{\partial x^2} = (-1) \frac{\partial}{\partial x}\left(f'(v \cdot t - x)\right) = (-1) \cdot f''(v \cdot t - x) \cdot (-1)$$

$$\frac{\partial^2 y}{\partial x^2} = f''(v \cdot t - x) \quad \longleftarrow$$

$$v^2 \cancel{f''(v \cdot t - x)} = \frac{F_0}{\rho A} \cancel{f''(v \cdot t - x)}$$

$$v^2 = \frac{F_0}{\rho A}$$

$\therefore$ $$v = \sqrt{\frac{F_0}{\rho A}}$$ $$y = f(v \cdot t - x)$$

→ Wave equation is satisfied

→ $$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

Harmonic Waves

$$y(x,t) = y_0 \sin\left(\frac{2\pi}{\lambda}(v \cdot t - x)\right)$$

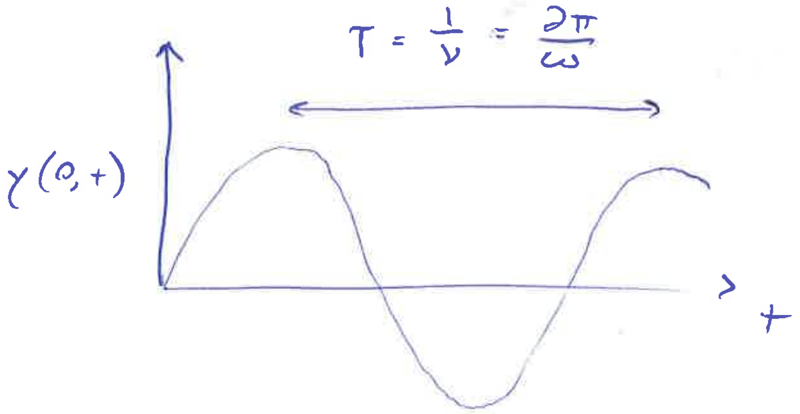
or

$$y(x,t) = y_0 \cos\left(\frac{2\pi}{\lambda}(v \cdot t - x)\right)$$

v : wave speed
λ : wavelength

$$y(x,t) = y_0 \sin\left(\frac{2\pi}{\lambda}(v \cdot t - x)\right)$$

$$y(0,t) = y_0 \sin\left(\frac{2\pi}{\lambda} v t\right)$$



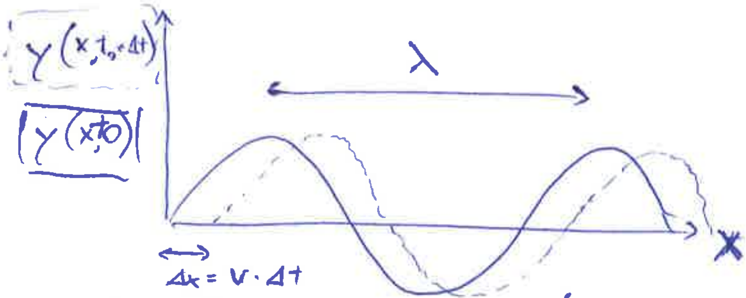
Period.

$$2\pi = \frac{2\pi}{\lambda} v T$$

$$T = \frac{\lambda}{v}$$

$$v = \lambda \cdot \frac{1}{T}$$

$$v = \lambda \cdot \nu$$



Exp: Hamster wheel model

$$y(x,t_0) = y_0 \sin\left(\frac{2\pi}{\lambda}(v \cdot t_0 - x)\right)$$

$$y(x,t_0+\Delta t) = y_0 \sin\left(\frac{2\pi}{\lambda}(v \cdot t_0 + \underbrace{v \cdot \Delta t} - x)\right)$$

$$\Delta x = v \cdot \Delta t$$

For $\Delta t = T$,

$$\Delta x = vT = v\frac{\lambda}{v} = \lambda$$

$$\Delta x = \lambda$$

$$y(x,t_0+T) = y_0 \sin\left(\frac{2\pi}{\lambda}(v \cdot t_0 - x) + 2\pi\right) = y_0 \sin\left(\frac{2\pi}{\lambda}(v t_0 - x)\right)$$

$$y(x,t_0+T) = y(x,t_0)$$

$$y(x,t) = y_0 \sin\left(\frac{2\pi}{\lambda}(v \cdot t - x)\right)$$

$$\frac{2\pi}{\lambda} v \cdot t = \frac{2\pi}{\lambda} \cdot \lambda \nu \cdot t = 2\pi \nu t = \omega t$$

$$-\frac{2\pi}{\lambda} \cdot x = -k x$$

$\therefore$

$$y(x,t) = y_0 \sin(\omega \cdot t - k x)$$

angular frequency ⟵ $\omega = \frac{2\pi}{\lambda} v = 2\pi\nu$

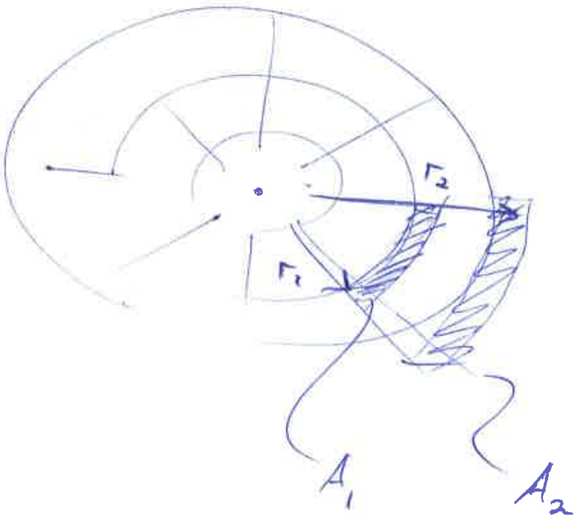
wave number ⟵ $k = \frac{2\pi}{\lambda}$

Wave speed & Wavelength

- Depends on material $v = \lambda \cdot \nu$
- Examples $\lambda = \frac{v}{\nu}$

Spherical Waves

$$y(r,t) = \frac{y_0}{r}\sin(\omega \cdot t - k \cdot r)$$



Power transmitted through A_1 (P_1) and power through A_2 (P_2) are the same, if the medium is not absorbing.

$$P_1 = P_2$$

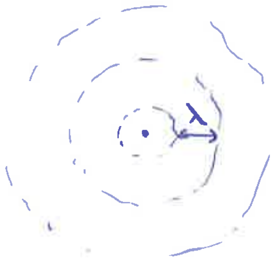
$$\frac{I_1}{I_2} = \frac{P_1/A_1}{P_2/A_2} = \frac{P_1}{A_1}\cdot\frac{A_2}{P_2} = \frac{A_2}{A_1}$$

Exp: Circular Wave in pool

$$\frac{I_1}{I_2} = \frac{A_2}{A_1} = \frac{r_2^{\,2}}{r_1^{\,2}} \longrightarrow I_2 = \frac{I_1 r_1^{\,2}}{r_2^{\,2}}$$

Doppler Effect

Stationary Source



$$v = \lambda \cdot \nu \qquad \nu = \frac{1}{T}$$

$$\lambda = \frac{v}{\nu} = vT$$

Moving Source:

$$\lambda_m = vT - v_r T = (v - v_r)T = \frac{v - v_r}{\nu}$$

$$\lambda_m = \frac{v}{\nu}\left(1 - \frac{v_r}{v}\right) = \lambda\left(1 - \frac{v_r}{v}\right)$$

$$\lambda_m = \frac{v}{\nu_m}$$

$$\nu_m = \frac{v}{\lambda_m} = \frac{v}{\lambda}\left(\frac{1}{1 - \frac{v_r}{v}}\right) = \frac{\nu}{1 - \frac{v_r}{v}}$$

$$\boxed{\nu_m = \frac{\nu}{1 - \frac{v_r}{v}}}$$

For $v_r << v$

$$\nu_m = \lim_{\frac{v_r}{v} \to 0} \frac{\nu}{1 - \frac{v_r}{v}} \qquad \text{let } x = \frac{v_r}{v}$$

$$\nu_m = \lim_{x \to 0} \frac{\nu}{1 - x} = \nu \underbrace{\left(\lim_{x \to 0} \frac{1}{1 - x}\right)}_{1 + x}$$

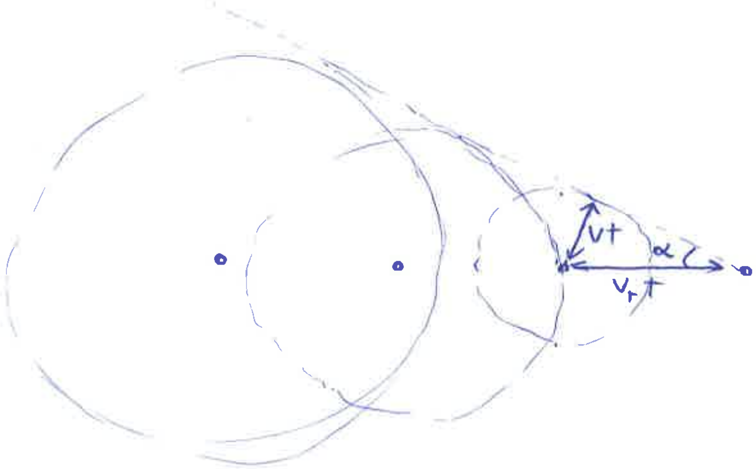
$$\nu_R = \nu(1 + x) = \nu\left(1 + \frac{v_r}{v}\right)$$

$$\underbrace{\nu_R - \nu}_{\Delta\nu} = \nu \frac{V_r}{V}$$

$$\boxed{\frac{\Delta\nu}{\nu} = \frac{V_r}{V}} \qquad \text{For } v_r << v$$

Exps : App, Auto, Wave pool

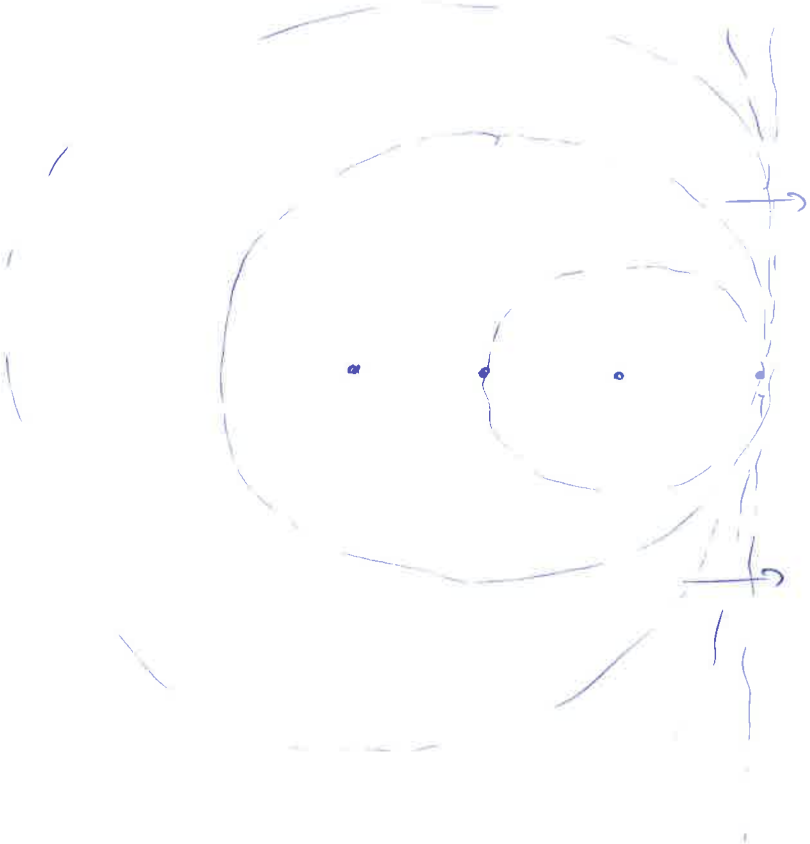
Sonic Boom



$$\sin\alpha = \frac{vt}{v_r t} = \frac{v}{v_r}$$

$$\sin\alpha = \frac{v}{v_r}$$

2α : angle of Mach cone



$v = v_r$

$\sin\alpha = 1$

$\alpha = 90°$

$2\alpha = 180°$