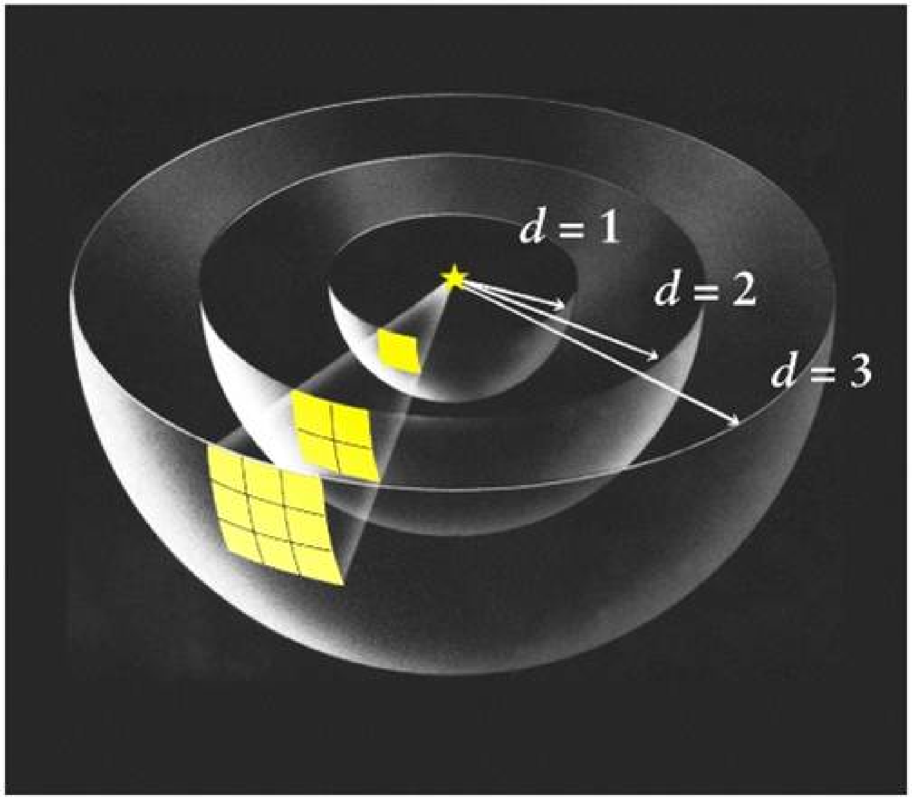
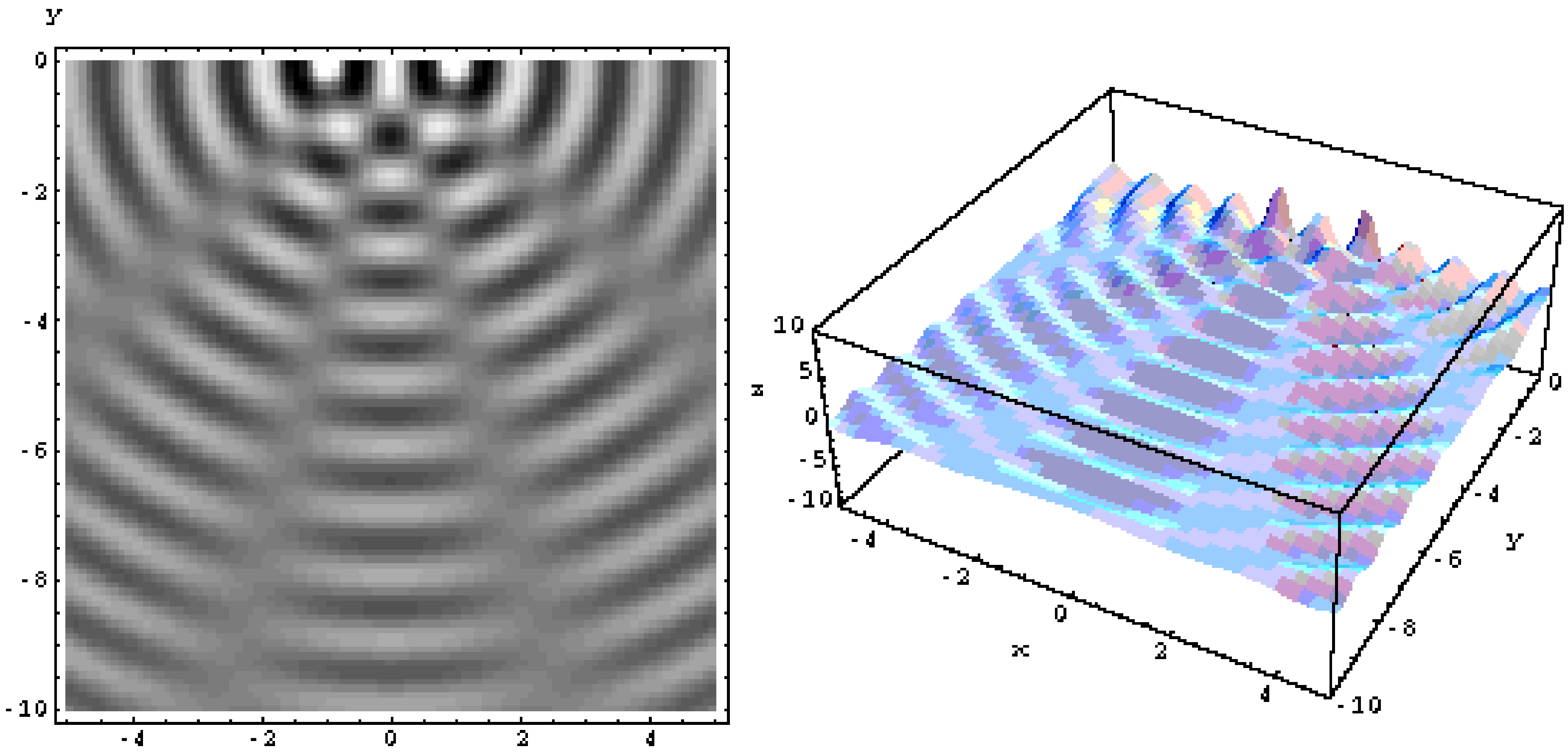
Introduction to Physics I

For Biologists, Geoscientists, & Pharmaceutical Scientists



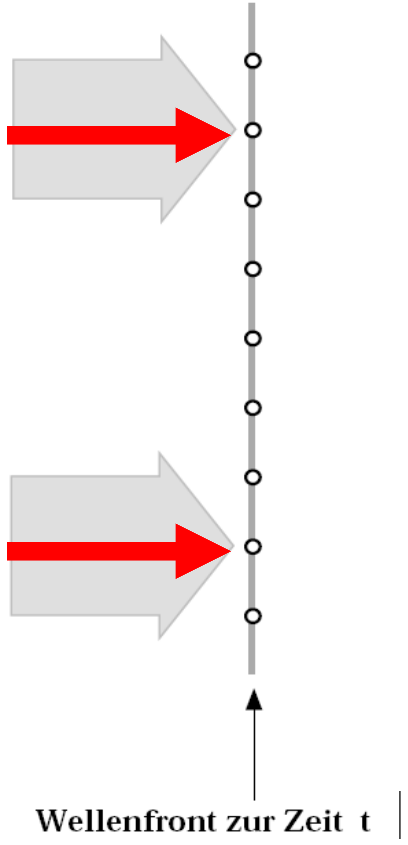
$d = 1$
$d = 2$
$d = 3$

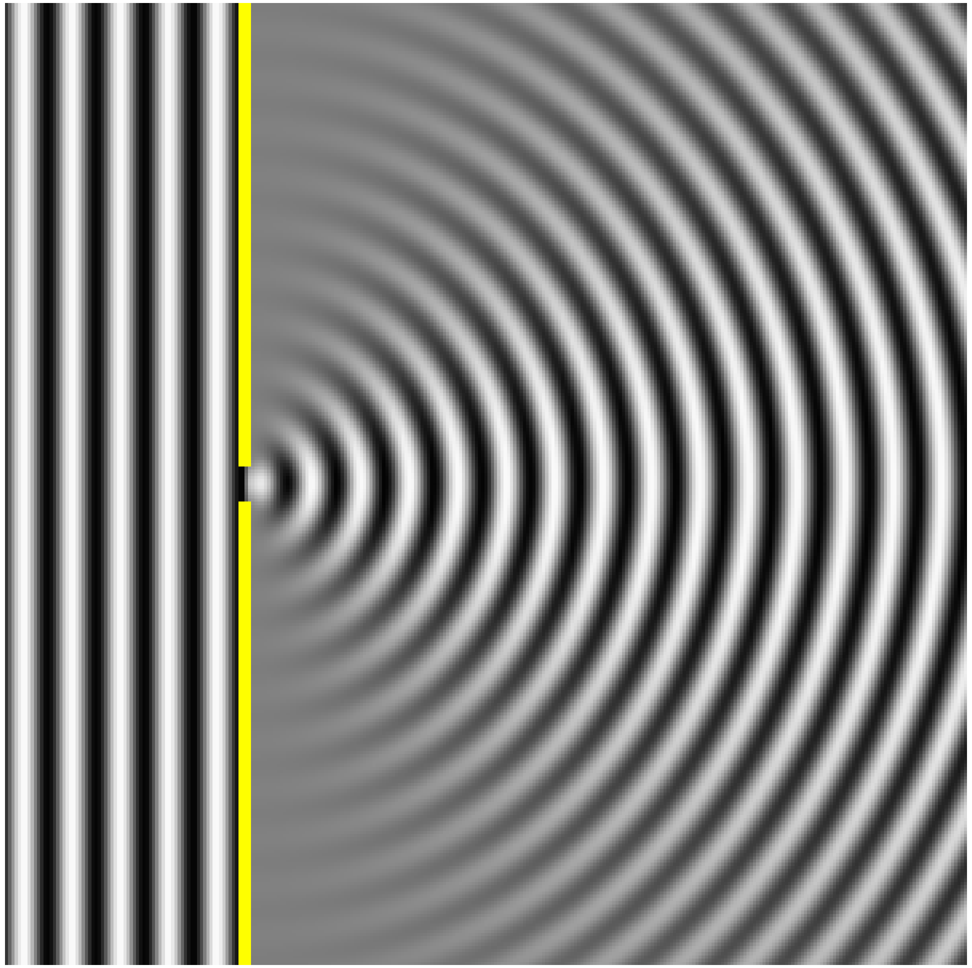


y
0
-2
-4
-6
-8
-10
-4
-2
0
2
4
10
5
z
0
-5
-10
-4
-2
0
x
2
4
0
-2
-4
y
-6
-8
-10

Beispiel: Ausbreitung einer ebenen Welle

Die Wellenfront zum Zeitpunkt t+Δt ergibt sich aus der Umhüllenden der Elementarwellen (Kugelwellen), die zum Zeitpunkt t ausgesandt wurden





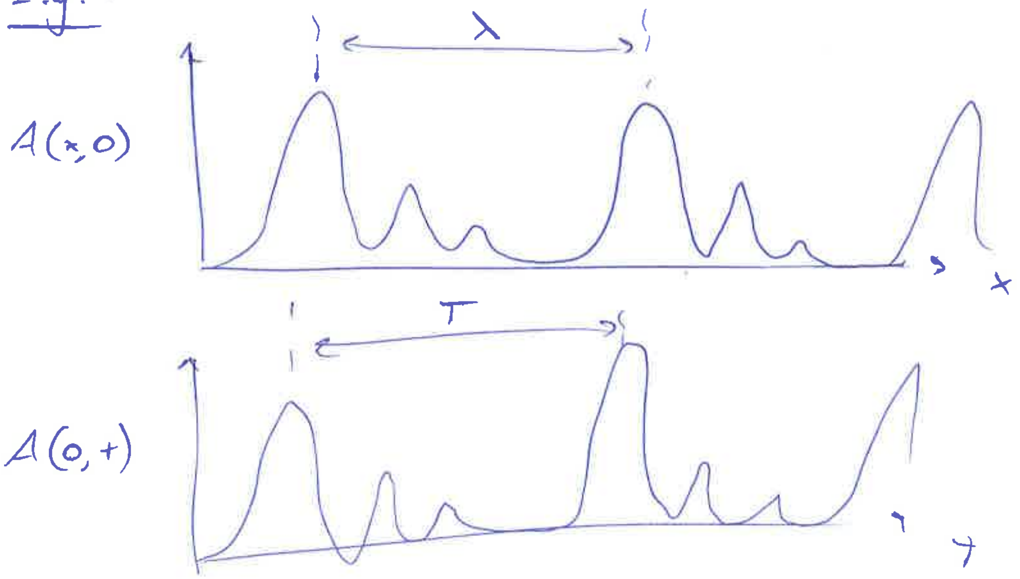
Waves II

Anharmonic waves

- Waves not described by simple sin or cos functions.

$$A(x,t) = A(x+\lambda, t) = A(x, t+T)$$

E.g.:



- Nevertheless such waves can be described by a Fourier series (a sum of harmonic waves):

$$A(x,t) = A_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t - nkx)$$
$$+ \sum_{n=1}^{\infty} b_n \cos(n\omega t - nkx)$$

Exp: Instruments

$n = 1$	Fundamental tone	1^{st} harmonic
$n = 2$	1^{st} over tone	2^{nd} harmonic
$n = 3$	2^{nd} overtone	3^{rd} harmonic
$n = 4$	3^{rd} overtone	4^{th} harmonic

In music : harmonic waves → Tones

periodic anharmonic waves → Complex tones

non-periodic waves → Noise

Superposition, Interference of Waves

$$Y_1 = Y_0 \sin(\omega t - kx - \phi_0)$$

$$Y_2 = Y_0 \sin(\omega t - kx)$$

$$Y = Y_1 + Y_2 = Y_0\left[\sin(\omega t - kx - \phi_0) + \sin(\omega t - kx)\right]$$

$$\sin(\alpha - \beta) + \sin(\alpha + \beta) = 2\cos\beta \sin\alpha$$

$$\alpha = \omega t - kx - \frac{\phi_0}{2}$$

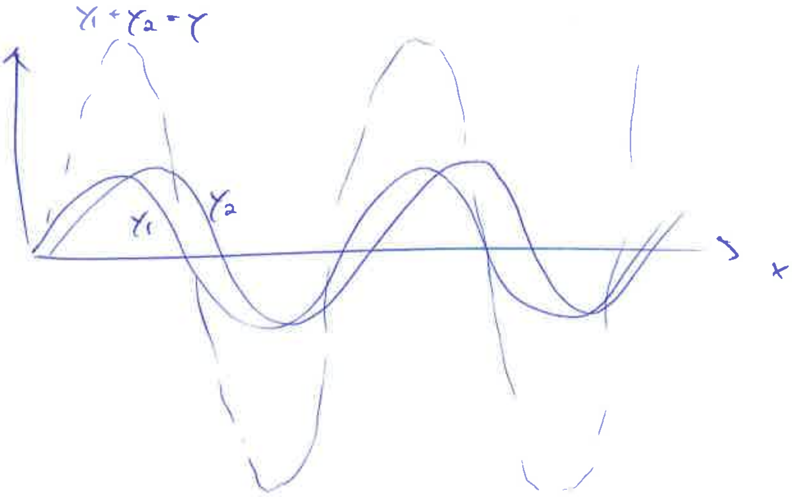
$$\beta = \frac{\phi_0}{2}$$

$$\therefore \quad Y = Y_0 \left(2 \cos\left(\frac{\phi_0}{2}\right) \sin\left(\omega t - kx - \frac{\phi_0}{2}\right) \right)$$

$$Y = \underbrace{2Y_0 \cos\left(\frac{\phi_0}{2}\right)}_{\text{Amplitude}} \underbrace{\sin\left(\omega t - kx - \frac{\phi_0}{2}\right)}_{\text{position \& time dependent}}$$

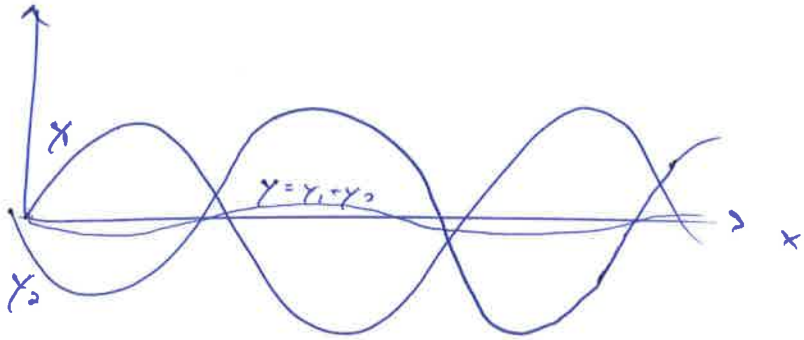
· Depending on ϕ_0 is either constructive or destructive interference possible.

For ϕ_0 near 0 :

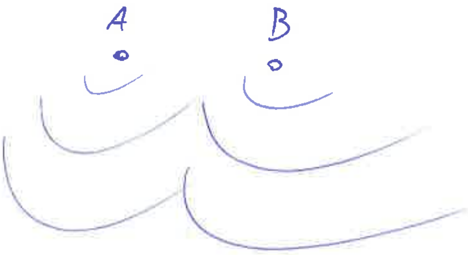


constructive interference

For ϕ_0 near π:

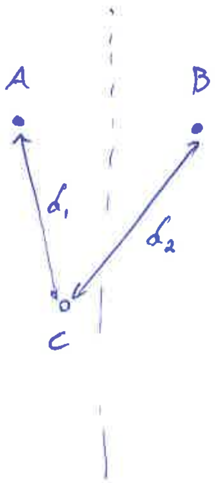


Exp: Interference in wave pool



$$y_C = y_0 \sin(\omega t - k d_1) + y_0 \sin(\omega t - k d_2)$$

Recall: $\sin(\alpha - \beta) + \sin(\alpha + \beta) = 2\cos\beta \sin\alpha$



$$\therefore \; y_C = 2\cos\left(\frac{k(d_1 - d_2)}{2}\right) \sin\left(\omega t - k\left(\frac{d_1 + d_2}{2}\right)\right)$$

Constructive Interference:

$$\frac{k(d_1 - d_2)}{2} = 0, \pi, 2\pi, \ldots$$

$$d_1 - d_2 = \frac{2n\pi}{k} = 0, \frac{2\pi}{k}, \frac{4\pi}{k}, \ldots$$

Destructive Interference:

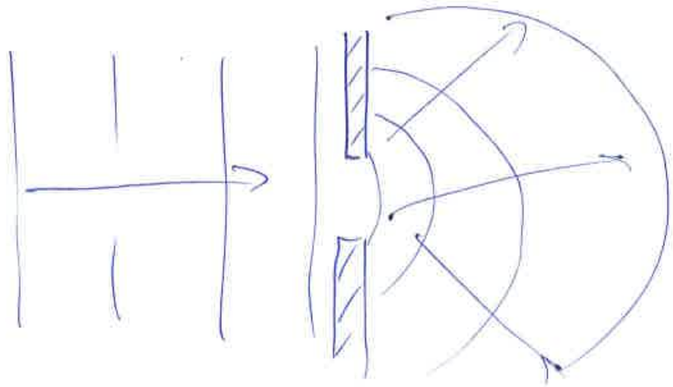
$$\frac{k(d_1 - d_2)}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$k(d_1 - d_2) = (2n+1)\pi$$

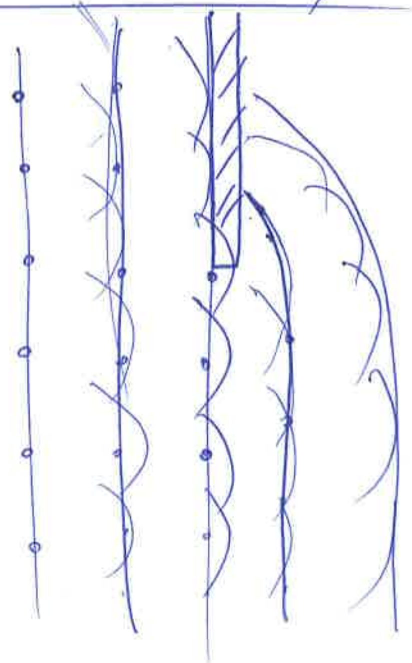
$$d_1 - d_2 = \frac{(2n+1)\pi}{k} = \frac{\pi}{k}, \frac{3\pi}{k}, \frac{5\pi}{k}, \ldots$$

Diffraction (Beugung)

- Diffraction describes the change in propagation direction of waves which encounter an obstacle.



Huygens-Fresnel Principle



Every point reached by a wave becomes a source of a spherical wave. The sum of these waves determines the form of the wave at subsequent times.

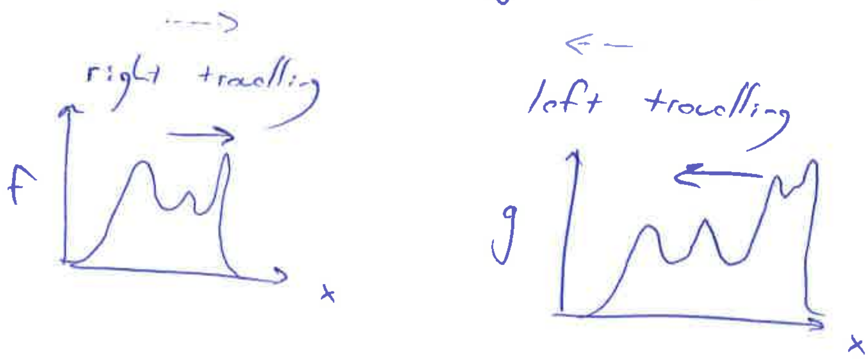
Exp: Wave pool!

Wave Reflection

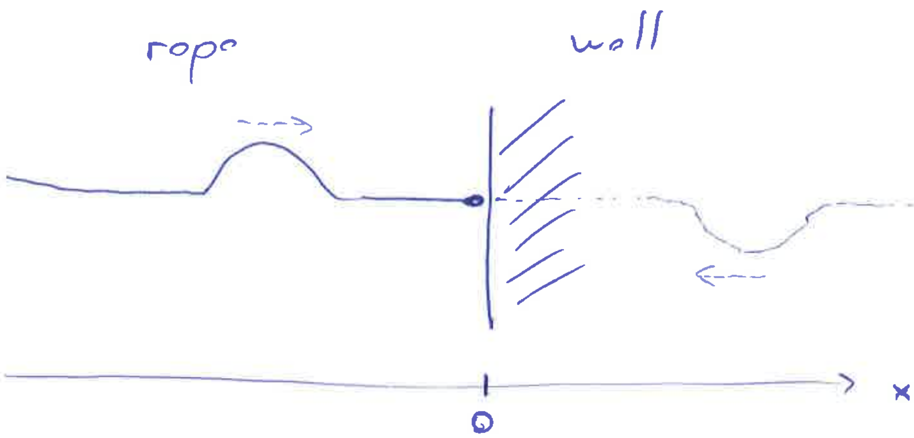
$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

This differential equation holds for functions of the following form:

$$y(x,t) = f(vt - x) + g(vt + x)$$



Example 1:



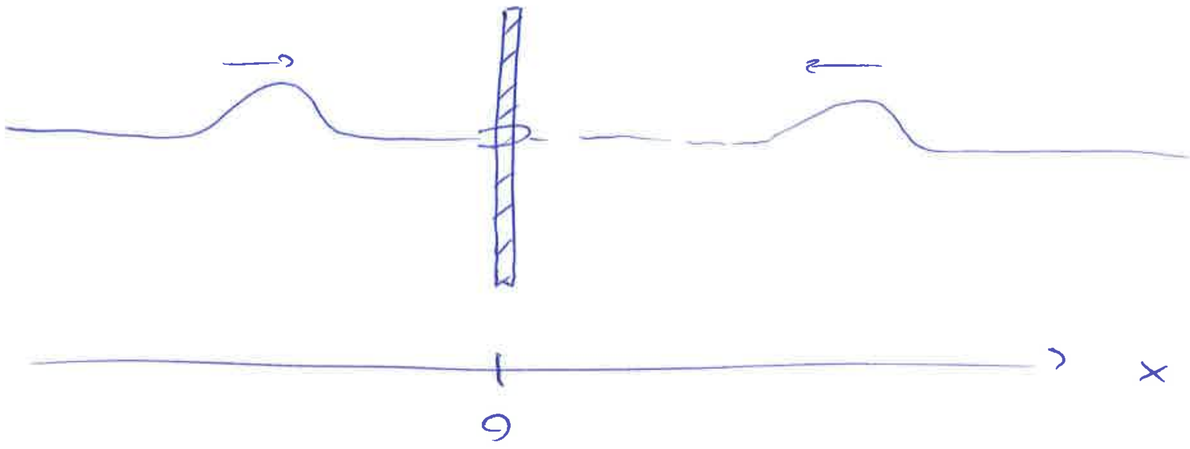
fixed to wall

$$y(0,t) = 0 = f(vt) + g(vt)$$

$$-f(vt) = g(vt)$$

$$\therefore\ y(x,t) = f(vt - x) - f(vt + x)$$

Example 2 :



$$F_y = F_0 \frac{dy}{dx}$$

post exerts no vertical force on rope

$$\frac{dy}{dx}(0,t) = 0 = \frac{d}{dx}\left(f(vt-x) + g(vt+x)\right)\Big|_{0,t}$$

$$0 = f'(vt)\cdot(-1) + g'(vt)\cdot(1)$$

$$f'(vt) = g'(vt)$$

$$f(vt) = g(vt)$$

$$\boxed{\therefore \quad y(x,t) = f(vt-x) + f(vt+x)}$$