Introduction to Physics I

For Biologists, Geoscientists, & Pharmaceutical Scientists

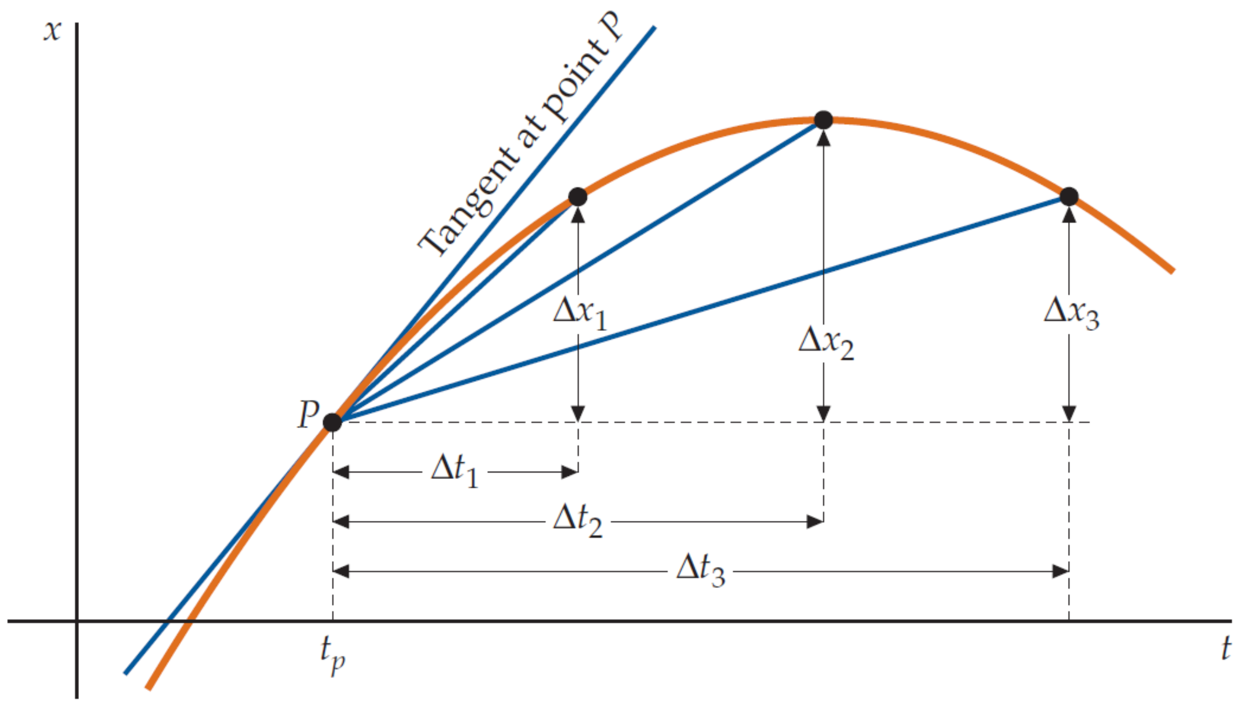
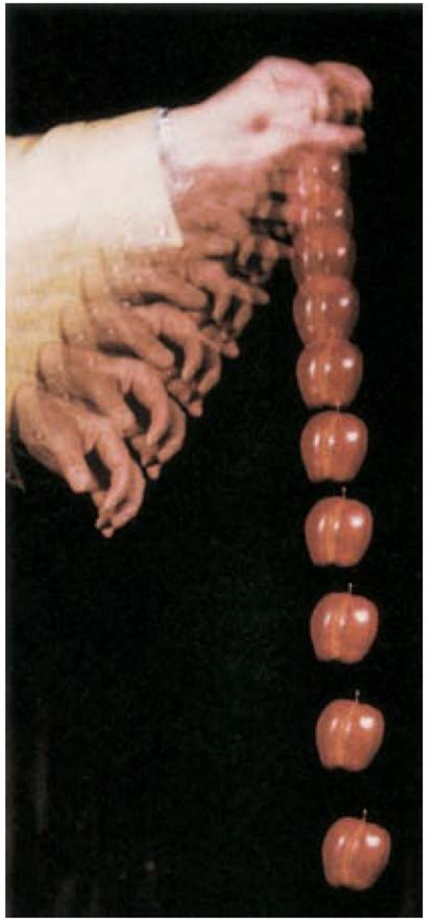


FIGURE 2-5 Graph of x versus t. Note the sequence of successively smaller time intervals, $\Delta t_3, \Delta t_2, \Delta t_1, \ldots$. The average velocity of each interval is the slope of the straight line for that interval. As the time intervals become smaller, these slopes approach the slope of the tangent to the curve at point t_P. The slope of this tangent line is defined as the instantaneous velocity at time t_P.

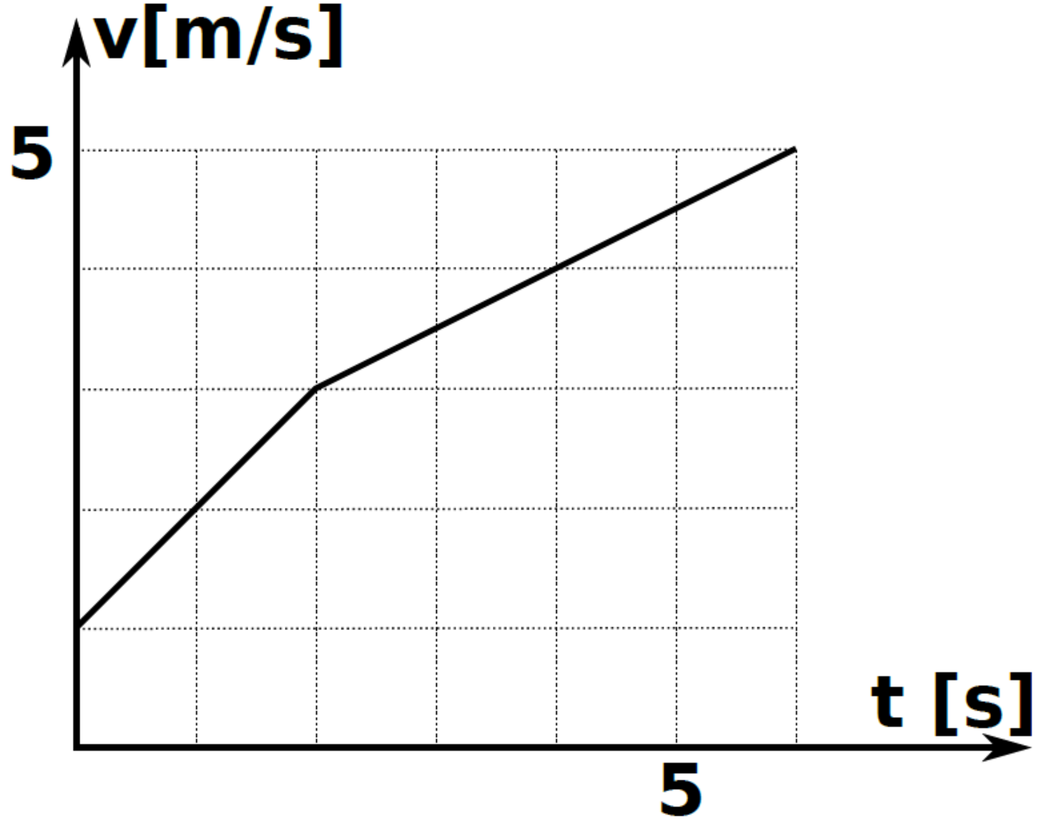
Tipler, Mosca, 6th ed.



A falling apple captured by strobe photography at 60 flashes per second. The acceleration of the apple is indicated by the widening of the spaces between the images. *(Estate of Harold E. Edgerton/Palm Press.)*

Tipler, Mosca, 6th ed.

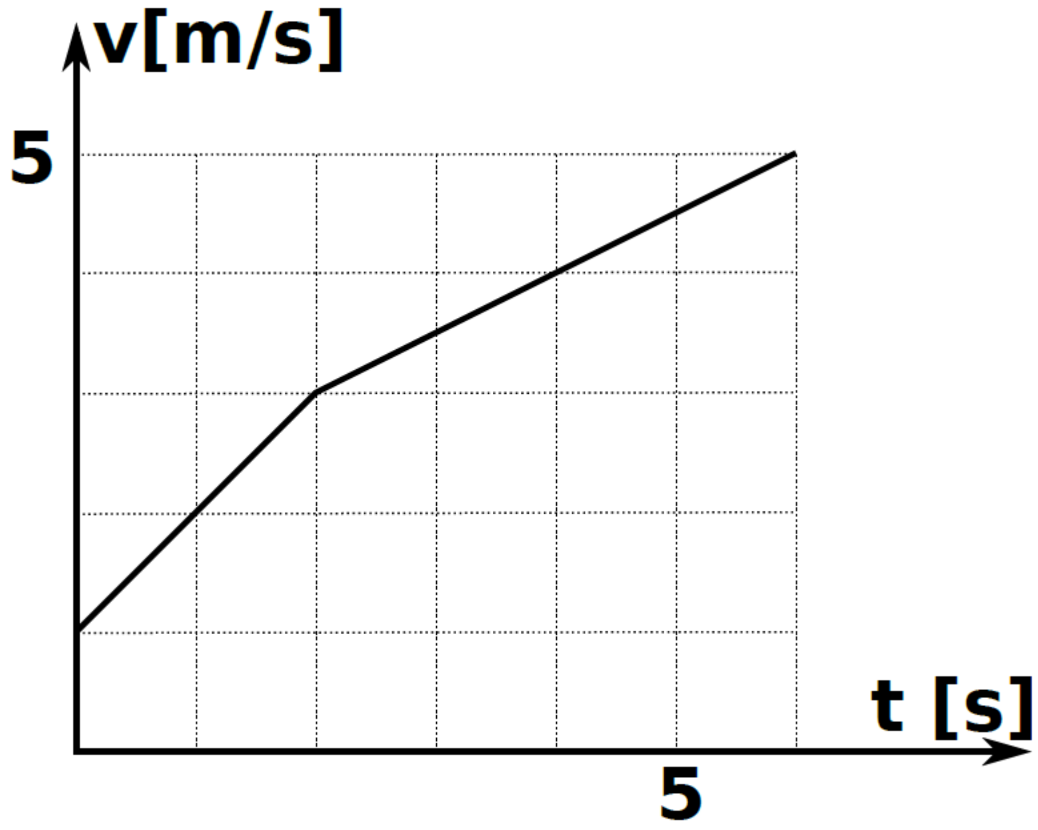
Frage: Geschwindigkeit & Beschleunigung



Der obige Graph zeigt die Geschwindigkeit eines Objekts in Abhängigkeit der Zeit. Wie gross ist die durchschnittliche Beschleunigung zwischen $t = 0s$ und $t = 6s$?

1. $3.0 m/s^2$
2. $1.5 m/s^2$
3. $0.83 m/s^2$
4. $0.67 m/s^2$
5. keine der Antworten.

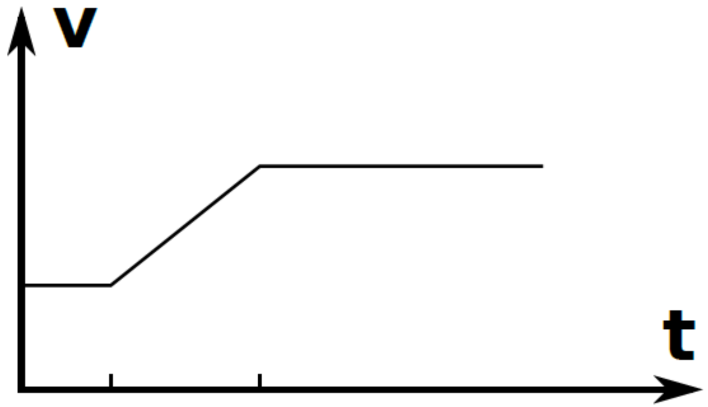
Frage: Geschwindigkeit & Beschleunigung



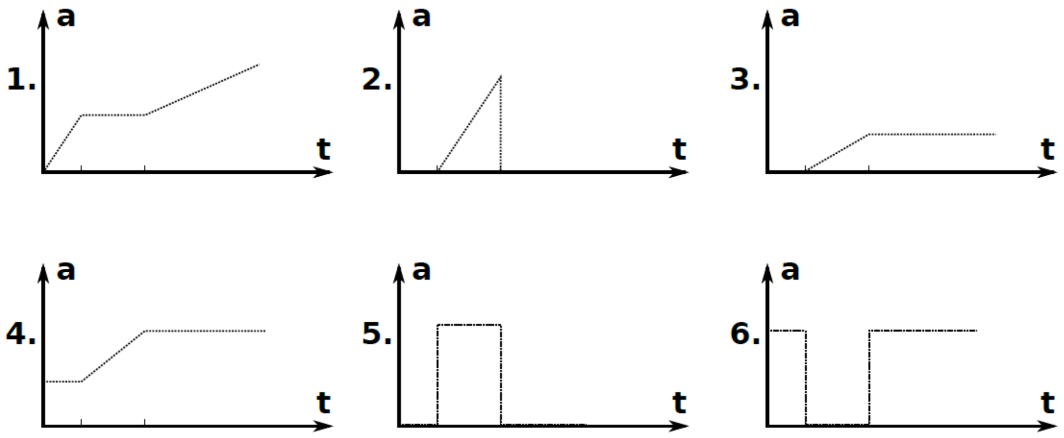
Antwort: 4. $0.67 m/s^2$

Die Geschwindigkeitsänderung (Beschleunigung) im gegebenen Zeitintervall beträgt $4 m/s$. Dies ergibt während $6 s$ eine Beschleunigung von $a = 0.67 m/s^2$.

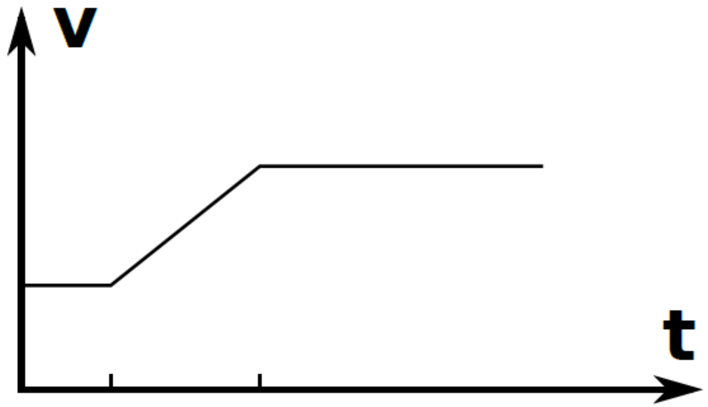
Frage: Geschwindigkeit & Beschleunigung

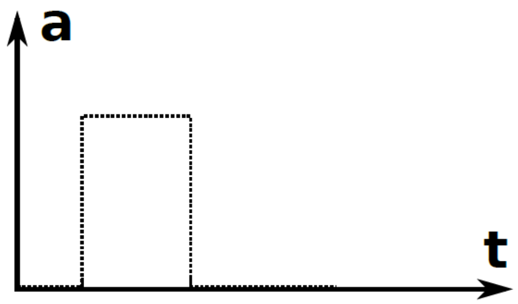


Ein Objekt bewegt sich mit einer Geschwindigkeit v, wie in obiger Skizze gezeigt. Welche der folgenden Graphen zeigt den richtigen qualitativen Verlauf der Beschleunigung?



Frage: Geschwindigkeit & Beschleunigung

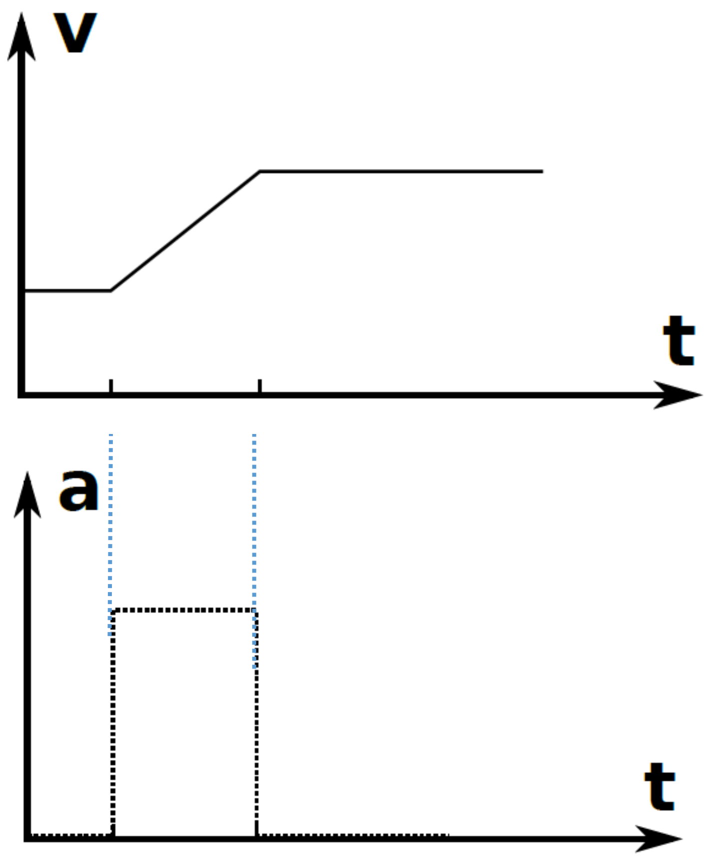


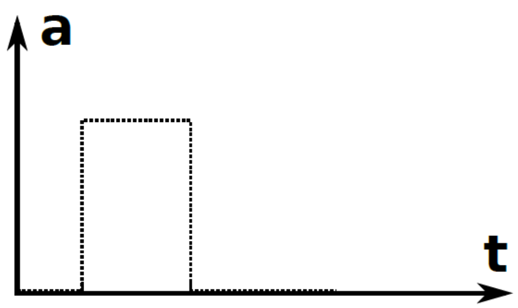


Antwort: 5.

Die Geschwindigkeit ändert sich zu Beginn nicht. Die Beschleunigung ist also null. Danach nimmt die Geschwindigkeit linear zu. Die Beschleunigung hat dann einen konstanten, positiven Wert. Zum Schluss ist die Änderung der Geschwingigkeit, und somit der Beschleunigung, wieder null.

Frage: Geschwindigkeit & Beschleunigung





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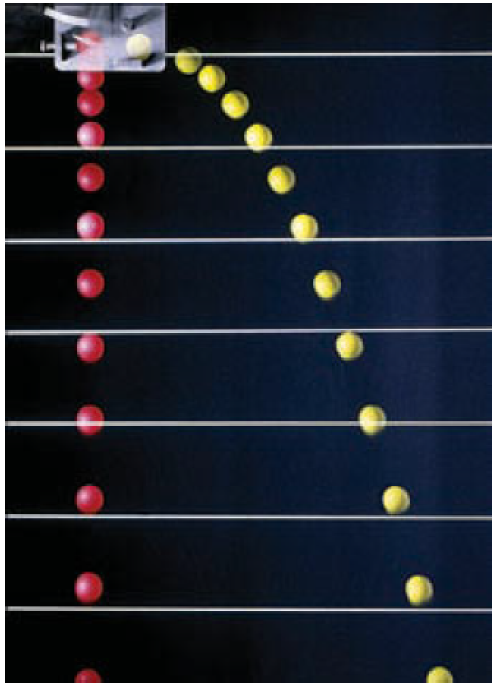


FIGURE 3-12 The red ball is released from rest at the instant the yellow ball rolls off the tabletop. The positions of the two balls are shown at successive equal time intervals. The vertical motion of the yellow ball is identical with the vertical motion of the red ball, thus demonstrating that the vertical motion of the yellow ball is independent of its horizontal motion. *(Richard Megna/Fundamental Photographs.)*

Tipler, Mosca, 6th ed.

Phys I ; L3 : Space and time

28.9.2017

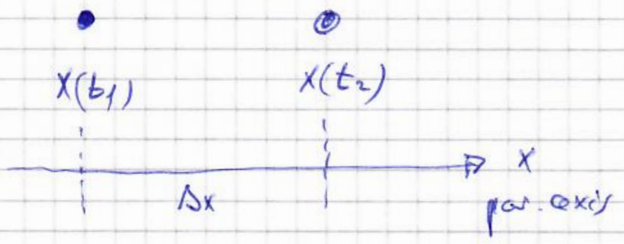
1

Motion in 1 dimension

$[x] = m$

$[t] = s$

- position $x(t)$



displacement $\Delta x = x(t_2) - x(t_1)$

during time interval $\Delta t = t_2 - t_1$

- average velocity (def) : (change of position)

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} \quad , \quad [v_{avg}] = \frac{m}{s}$$

- instantaneous velocity (def) {

(slide) Tipler, Fig 2.5 : show, v is the slope of the line tangent to the x(t) curve

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \dot{x}$$

notation , derivative of x with respect to t

- average acceleration (def) (change of speed)

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v(t_2) - v(t_2)}{t_2 - t_1}$$

- instantaneous acceleration

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \ddot{x}$$

notation

later

~~(slide) Tipler, Fig follows apple est...~~

~~(Slides) 2 Fragen : Geschw & Beschleunigung~~

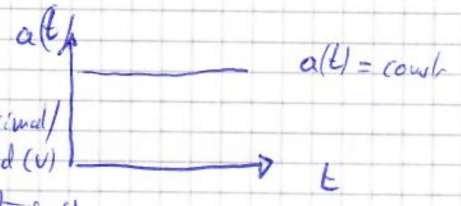
- motion with constant acceleration:

$a = \text{const}$

$a = \frac{dv}{dt}$

dv: small (infinitesimal) change of speed (v)

dt: small (infinitesimal) change of time (t)



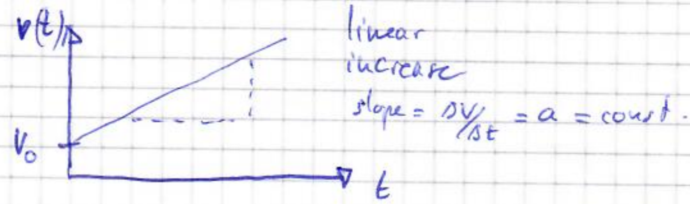
$a \cdot dt = dv$

$$\int_0^t a \cdot dt = \int_{v(0)}^{v(t)} dv$$

$$a \cdot t - a \cdot 0 = v(t) - v(0)$$

$$a \cdot t = v(t) - v_0$$

and $$\boxed{v(t) = a \cdot t + v_0}$$



Speed: $v(t) = \frac{dx(t)}{dt}$ $\quad (v = v(t))$

$$v \cdot dt = dx$$

$$\int_0^t v \cdot dt = \int_{x(0)}^{x(t)} dx$$

$$\int_0^t (a \cdot t + v_0) \cdot dt = \int_{x_0}^{x(t)} dx \quad , x_0 = x(t=0)$$

$$\frac{1}{2} a t^2 + v_0 \cdot t = x(t) - x_0$$

and $$\boxed{x(t) = \frac{1}{2} a t^2 + v_0 \cdot t + x_0}$$

(Slide) Tipler, falling apple: visualize acceleration

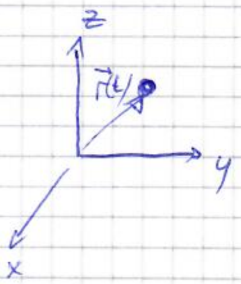
def: motion with v = const ≝ uniform motion

(Slide) 2 questions, velocity, acceleration

Motion in two and three dimensions

$x(t)$ → $\vec{r}(t) = (x(t), y(t), z(t))$

position → position vector



here: cartesian coordinates
(also cylindrical, spherical, ...)

1D: $\Delta x = x(t_2) - x(t_1)$ $\overset{3D}{\Rightarrow}$ $\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1) = \left(x(t_2) - x(t_1);\ y(t_2) - y(t_1);\ z(t_2) - z(t_1)\right)$

displacement vector

Velocity: $\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \left(\frac{dx(t)}{dt};\ \frac{dy(t)}{dt};\ \frac{dz(t)}{dt}\right)$

$= \left(v_x(t);\ v_y(t);\ v_z(t)\right)$

Notes: avg velocity vector $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \left(\frac{x(t_2) - x(t_1)}{t_2 - t_1};\ \dots;\ \right)$

instantaneous velocity vector
≡ velocity $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

magnitude of velocity vector: $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

similarly, for the acceleration,
(instantaneous)

$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{r}(t)}{dt^2} = \left(\frac{d^2x(t)}{dt^2};\ \frac{d^2y(t)}{dt^2};\ \frac{d^2z(t)}{dt^2}\right)$

$= \left(a_x(t);\ a_y(t);\ a_z(t)\right)$

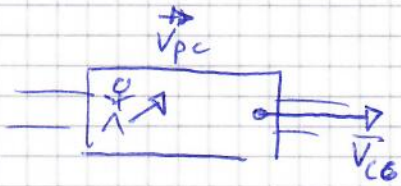
relative velocity and reference frame

travel in train or airplane : what is your velocity

$v_1 = 0$ relative to train/airplane (if seated)
$v_2 \neq 0$ " to earth surface
$v_3 \neq 0$ " " " " and $v_3 \neq v_2$ if in a jet stream

→ a velocity makes sense only when specifying the reference frame.

example : person ~~walking~~ in (~~train~~) railroad car :
(P) (C)



$\vec{v}_{pc}$: velocity of person relative to car

v_{CG} : " " car " to ground

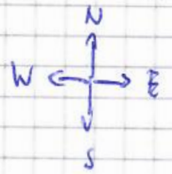
Velocity of person relative to ground :

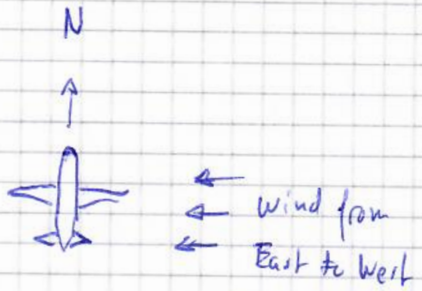
$$\vec{v}_{pG} = \vec{v}_{pc} + \vec{v}_{CG} \quad \text{(additivity)}$$

note $\vec{v}_{pc} = -\vec{v}_{cp}$

↑ velocity of car relative to person

<u>example</u> plane + wind (Tipler, example 3-2)





Plane: $|\vec{V}_{pA}| = 600$ km/h, airspeed of plane

Wind: $|\vec{V}_w| = |\vec{V}_{AG}| = 100$ km/h, wind speed (air to ground speed)

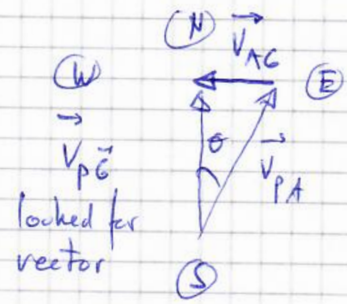
a) Pilot wants to fly North. In which direction shall the plane head?

b) What is the ground-speed of the plane?

- Wind => plane drifts off course towards West.

a). velocity of plane relative to ground: $\vec{V}_{pG} = \vec{V}_{pA} + \vec{V}_{AG}$

- vector addition diagram:
  (Wind direction is fixed w respect to ground)



|| $\sin\theta = \dfrac{V_{AG}}{V_{pA}} = \dfrac{100}{600}$ $\theta \simeq 10^0$ (9.59°)

(note: $V_{AG} = |\vec{V}_{AG}|$)

|| Plane shall fly with 10° angle East to North

b) $V_{pA}^2 = V_{pG}^2 + V_{AG}^2$ (triangle with ⊡ angle)

$V_{pG} = \left(V_{pA}^2 - V_{AG}^2\right)^{1/2} \cong 592$ km/h

Note if plane would fly due East $V_{pG} = 500$ km/h

non uniform motion: adding acceleration ($a \neq 0$, $a = \frac{dv}{dt}$)

28.9

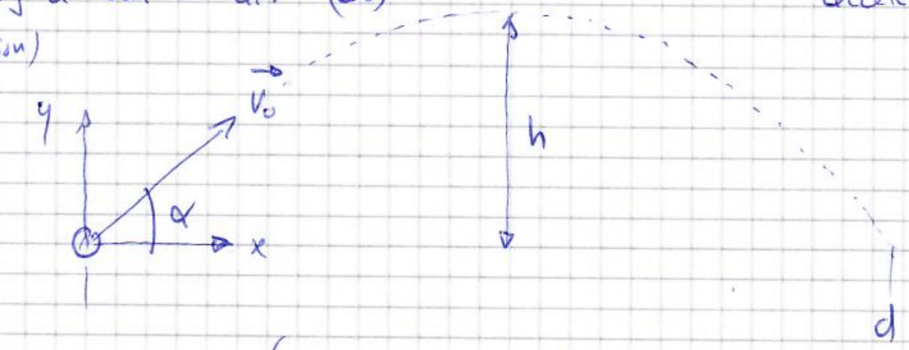
Seen eq. of motion (10):

$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0 \qquad (1)$$

$$v(t) = a \cdot t + v_0 \qquad (2)$$

launching a ball in air (20) (no friction)

acceleration: $\downarrow \vec{g}$ (gravity)



$\vec{v}_0 = (v_{0x}, v_{0y}) = (v_0 \cdot \cos\alpha, v_0 \cdot \sin\alpha)$

$v_0 = 10$ m/s, $\alpha = 60°$

$\vec{a} = (0, -g)$, $g = 9.81$ m/s^2

Find $\vec{r}(t)$ and $\vec{v}(t)$ for $t = 1$ s

a) initial velocity vector : $t=0$ $\quad \vec{v}_0 = (v_0 \cdot \cos\alpha, v_0 \sin\alpha)$
$= (5, 8.66)$

b) velocity at time t: $\quad \vec{v}(t) = (v_x(t), v_y(t))$

$$v_x(t) = v_{0x} + \int_0^t a_x dt = v_{0x}$$

(↑ Eq. 2 above)

$$v_y(t) = v_{0y} + \int_0^t a_y dt = v_{0y} - g\int_0^t dt = v_{0y} - g \cdot t$$

and $\vec{v}(t) = (v_{0x}, v_{0y} - g \cdot t)$

c) position at time t: $\quad \vec{r}(t) = (x(t), y(t))$, $dx = x_0 + v_x dt$

$$x(t) = x_0 + \int_0^t v_x dt \quad \text{~~+ (½ a_x t, const acceleration)~~}$$

$$= v_{0x} \cdot t \qquad (x_0 = 0, y_0 = 0)$$

$$y(t) = y_0 + \int_0^t v_y(t) \cdot t = y_0 + \int_0^t (v_{0y} - g \cdot t) \cdot dt$$

$$= v_{0y} \cdot t - \frac{1}{2}gt^2$$

$\vec{r}(t) = (v_{0x} \cdot t, \; v_{0y} \cdot t - \frac{1}{2}gt^2)$

28.9

6'

Exp Waterjet.

- project parabola on wall
- adjust waterjet, show overlap with parabolic curve

Exp schiefer Wurf + Platte

explain trajectory, then equation

ask assumptions!?

d) for $t = 1\,sec$

$$\vec{r} = (v_{ox} \cdot t \,,\; v_{oy} \cdot t - \tfrac{1}{2} g t^2)$$
$$= (5 \cdot 1 \,,\; 8.66 \cdot 1 - \tfrac{1}{2} \cdot 9.81 \cdot 1^2) = (5\,m \,,\; 3.76\,m)$$

$$\vec{v} = (v_{ox} \,,\; v_{oy} - g \cdot t)$$
$$= (5\,m/s \,,\; -1.15\,m/s)$$ going right and down

(Slide) Tipler Fig 3-12 : effect $v_{ox} \neq 0$
note, vertical motion is identical for both balls
i.e., vertical motion indep. of horizontal motion (here)

With eq. of motion, calculate precisely trajectory

(Slide) Cassini mission : include effect of planets, sun (additional acceleration effect)

<u>So far</u> :
- the origin of the acceleration has not been discussed (gravitation : force due to earth mass)
- we assumed no other disturbances (forces) acting on motion of objects considered (no friction, no electrical forces)

Exp Freier Fall quantitativ:

- ball falling from ceiling, meas. g

$$x(t) = \frac{1}{2} a t^2 + v_0 t + x_0$$

$v_0 = 0 \qquad x_0 = 4.92\,m$

$$-4.92 = \frac{1}{2} \cdot (+a) \cdot t^2 \quad , \quad t \sim 1s$$

$$\Rightarrow a \cong -9.84 \frac{m}{s^2} \quad , \text{ gravitation}$$

Exp freier fall, qualitativ

plume and ball in tube a) air

b) no air