

Introduction to Physics I

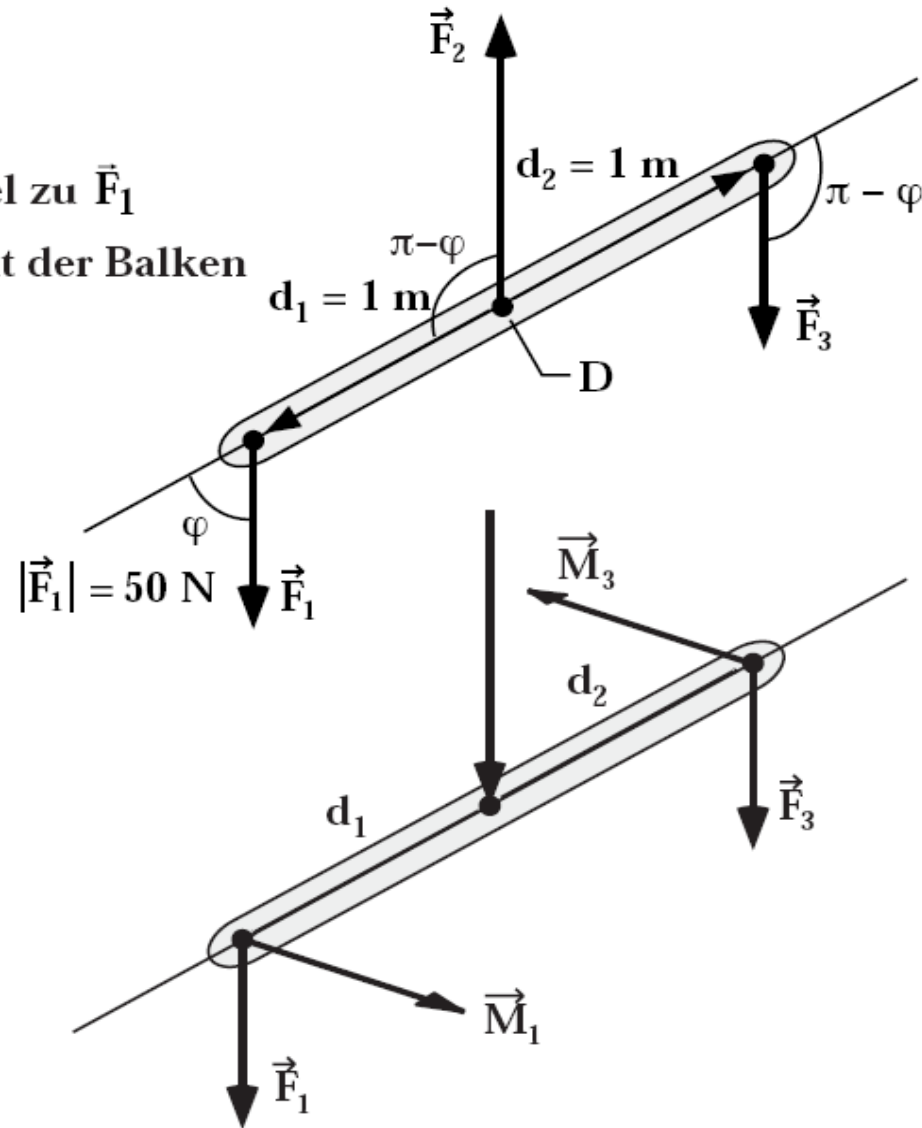
For Biologists, Geoscientists, & Pharmaceutical Scientists

Die Drehmomente der Kräfte \vec{F}_i können bezüglich einer beliebigen Drehachse berechnet werden.

Beispiel:

Gegeben: $\vec{F}_1, d_1, d_2, \vec{F}_2$ und \vec{F}_3 sind parallel zu \vec{F}_1

Wie gross müssen $|\vec{F}_2|$ und $|\vec{F}_3|$ sein, damit der Balken im Gleichgewicht ist?



$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

$$\vec{F}_2 + \vec{F}_3 = -\vec{F}_1$$

wählen Drehpunkt D:

$$\vec{M}_1 + \vec{M}_2 + \vec{M}_3 = 0$$

Beträge:

$$d_1 F_1 \sin \varphi - d_2 F_3 \sin(\pi - \varphi) = 0$$

$$F_3 = \frac{d_1}{d_2} F_1$$

$$\rightarrow \vec{F}_2 = -\left(1 + \frac{d_1}{d_2}\right) \vec{F}_1 = -2\vec{F}_1 = \mathbf{-100\text{ N}}, \quad \vec{F}_3 = \vec{F}_1 = \mathbf{50\text{ N}}$$

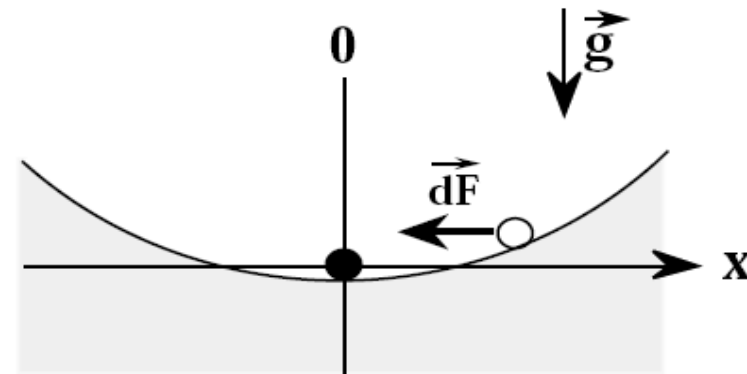
103-12

3. Dynamik

3 Arten von Gleichgewicht

1. Stabiles Gleichgewicht

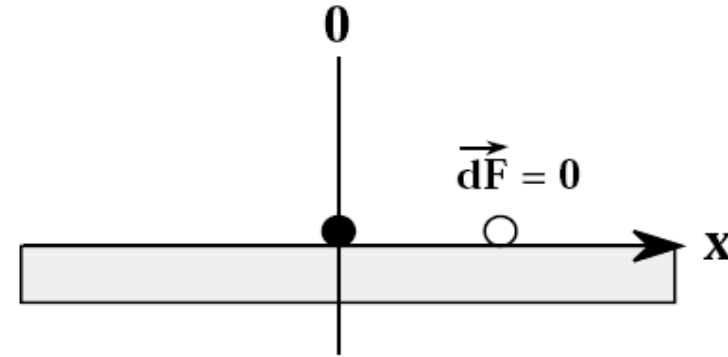
Modell



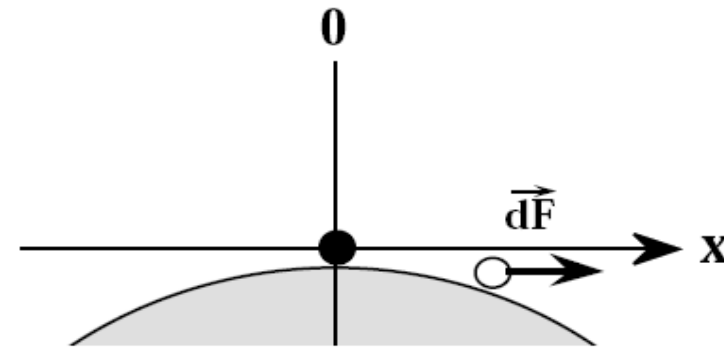
Gleichgewichtslage $x = 0$

Auslenkung um dx bewirkt eine rücktreibende Kraft \vec{F}

$$dF = -\alpha dx$$

2. indifferentes Gleichgewicht

es existieren unendlich viele Gleichgewichtslagen. Eine Auslenkung um Δx bewirkt keine Kraftwirkung.

3. labiles Gleichgewicht

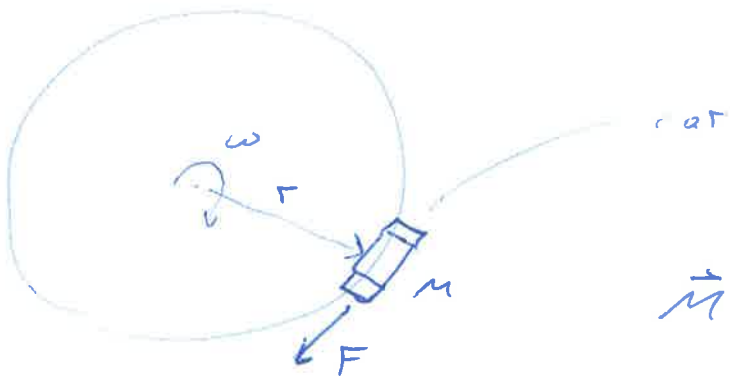
Gleichgewichtslage $x = 0$

Auslenkung um dx bewirkt eine Kraft in Richtung dx : $dF = \alpha dx$

Neben diesen drei Gleichgewichtsarten existiert noch das metastabile Gleichgewicht.

Torque (Drill moment)

Car driving on circular track.



$$\vec{M} = \vec{r} \times \vec{F}$$

$$M = rF = rma = rm \frac{dv}{dt}$$

$$M = rm \frac{d(\omega r)}{dt}$$

$$M = mr^2 \frac{d\omega}{dt}$$

or with vectors ...

$$\vec{M} = \vec{r} \times \vec{F} = \vec{r} \times m\vec{a} = \vec{r} \times m \frac{d\vec{v}}{dt}$$

$$\vec{M} = \vec{r} \times m \frac{d}{dt} (\vec{\omega} \times \vec{r})$$

$$\vec{M} = mr^2 \frac{d\vec{\omega}}{dt}$$

①

$$\vec{p} = m\vec{v}$$

$$\vec{M} = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \frac{d(m\vec{v})}{dt}$$

$$= \vec{r} \times m \frac{d\vec{v}}{dt} \quad \vec{v} = \vec{\omega} \times \vec{r}$$

$$= \vec{r} \times m \frac{d(\vec{\omega} \times \vec{r})}{dt}$$

$$= \vec{r} \times m \left(\frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt} \right)$$

$$= \vec{r} \times m \left(\frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \vec{v} \right)$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\vec{A} \times (\vec{B} \times \vec{C}) =$$

$$(\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$$

$$\vec{\omega} \cdot \vec{r} \vec{\omega} - (\vec{\omega} \cdot \vec{\omega})\vec{r}$$

$$= \vec{r} \times m \left(\frac{d\vec{\omega}}{dt} \times \vec{r} - (\vec{\omega} \cdot \vec{\omega})\vec{r} \right)$$

$$= m \left(\vec{r} \times \left(\frac{d\vec{\omega}}{dt} \times \vec{r} \right) - \omega^2 \vec{r} \right)$$

$$= m \left[(\vec{r} \cdot \vec{r}) \frac{d\vec{\omega}}{dt} - \vec{r} \cdot \frac{d\vec{\omega}}{dt} \vec{r} \right]$$

$$\vec{M} = m r^2 \frac{d\vec{\omega}}{dt}$$

Translation

$$\vec{F} = m \frac{d\vec{v}}{dt}$$

mass

acceleration

↳ 2nd Law

Rotation

$$\vec{M} = \overset{J}{mr^2} \frac{d\vec{\omega}}{dt}$$

moment
of
inertia

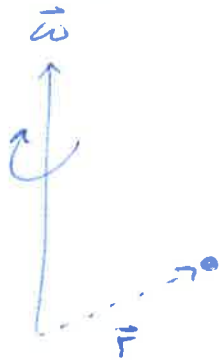
angular
acceleration

2nd Law For

Rotation

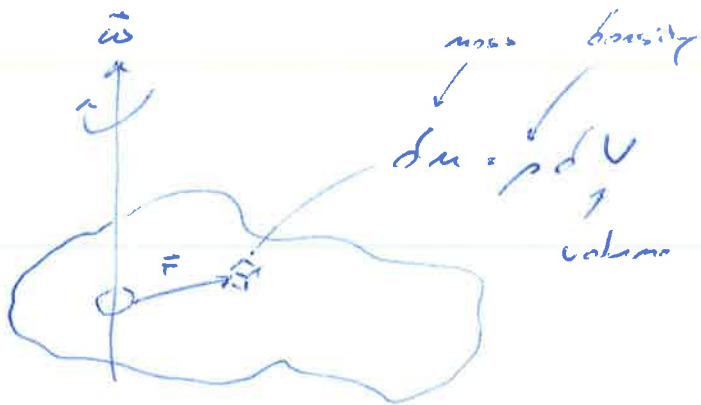
Moment of Inertia (Trägheitsmoment)

Point mass:



$$J = m r^2$$

Continuous Object:

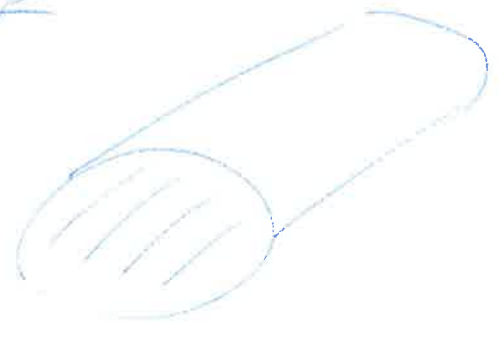


$$dJ = r^2 d\mu = r^2 \rho dV$$

$$\text{Total mass: } M = \int_B d\mu = \int_B \rho dV$$

$$\text{Total moment of inertia: } J = \int_B dJ = \int_B r^2 d\mu = \int_B r^2 \rho dV$$

Expt



Filled Cylinder

M_1

J_1



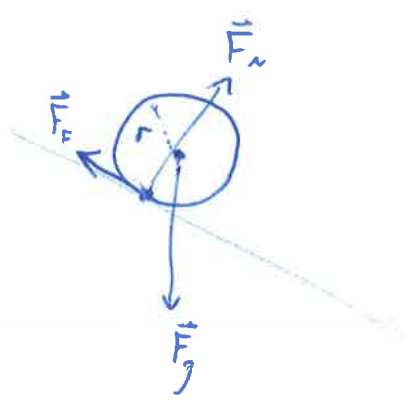
Hollow Cylinder

M_2

J_2

$\rightarrow M_1 = M_2$, $\int_{B_1} dm = \int_{B_2} dm$

$\rightarrow J_1 \neq J_2$, $\int_{B_1} r^2 dm \neq \int_{B_2} r^2 dm$



$\vec{M} = \vec{r} \times (\vec{F}_N + \vec{F}_g + \vec{F}_F)$

$\vec{M} = r F_F$

$M = J \frac{d\omega}{dt} = r F_F$

$\frac{d\omega}{dt} = \frac{r F_F}{J}$

Angular Momentum (Drehimpuls)

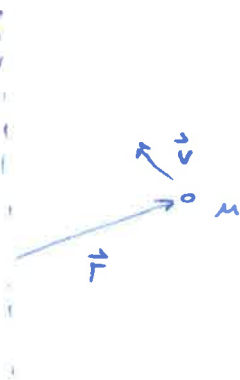
Momentum (Impuls) : $\vec{p} = m \vec{v}$

$\vec{L} = \vec{r} \times \vec{p}$ ← Angular Momentum

$\vec{L} = \vec{r} \times m \vec{v}$

For circular motion :

$\vec{v} = \vec{\omega} \times \vec{r}$



$\vec{L} = \vec{r} \times m (\vec{\omega} \times \vec{r})$

$\vec{L} = m \vec{r} \times (\vec{\omega} \times \vec{r}) = m (\vec{r} \cdot \vec{r}) \vec{\omega} - (\vec{r} \cdot \vec{\omega}) \vec{r}$

$\vec{L} = \underbrace{m r^2}_{J} \vec{\omega}$

$\vec{A} \cdot (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C}) \vec{B} - (\vec{A} \cdot \vec{B}) \vec{C}$

$\vec{L} = J \vec{\omega}$

moment of inertia angular velocity

Rotation

$\vec{p} = m \vec{v}$

mass velocity

Translation

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$\frac{d\vec{L}}{dt} = \vec{v} \times (m\vec{v}) + \vec{r} \times \vec{F}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{\tau}}$$

rotation

$$\boxed{\frac{d\vec{p}}{dt} = \vec{F}}$$

translation

Translation

$$F$$

$$\vec{v} = \dot{x}$$

$$\vec{a} = \dot{v} = \ddot{x}$$

$$m$$

$$\vec{p} = m \vec{v}$$

$$F = \dot{p}$$

Rotation

$$\phi$$

$$\omega = \dot{\phi}$$

$$\dot{\omega} = \ddot{\phi}$$

$$J$$

$$\vec{L} = \cancel{m r^2} J \cdot \vec{\omega}$$

$$\vec{\tau} = \dot{\vec{L}}$$

Equilibrium (Gleichgewicht)

A body is in equilibrium (not accelerating or rotationally accelerating) when the sum of all forces and the sum of all torques is zero.

$$\rightarrow \vec{F}_{\text{tot}} = \sum_{i=1}^N \vec{F}_i = 0$$

$$\frac{d\vec{p}_{\text{tot}}}{dt} = 0$$

$$\frac{d}{dt} (m_{\text{tot}} \vec{v}_{\text{tot}}) = 0$$

$$m_{\text{tot}} \frac{d\vec{v}_{\text{tot}}}{dt} = 0$$

$$m_{\text{tot}} \vec{a}_{\text{tot}} = 0 \rightarrow \vec{a}_{\text{tot}} = 0$$

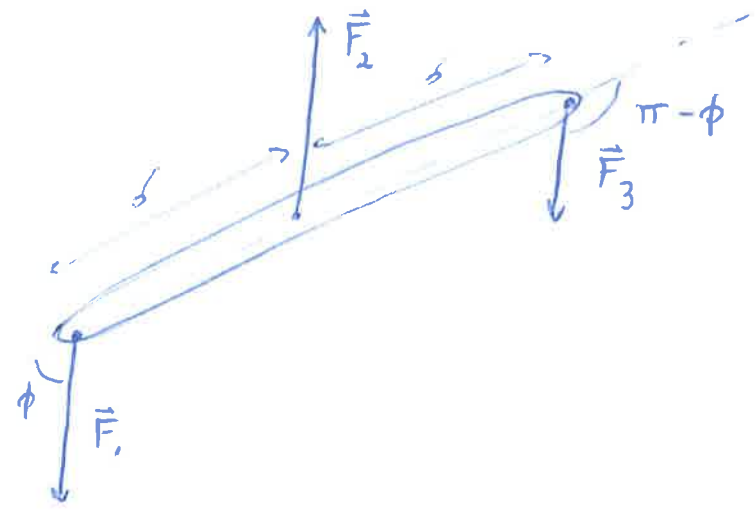
$$\rightarrow \vec{M}_{\text{tot}} = \sum_{i=1}^N \vec{M}_i = 0$$

$$\frac{d\vec{L}_{\text{tot}}}{dt} = 0$$

$$\frac{d}{dt} (J_{\text{tot}} \vec{\omega}_{\text{tot}}) = 0$$

$$J_{\text{tot}} \frac{d\vec{\omega}_{\text{tot}}}{dt} = 0 \rightarrow \frac{d\vec{\omega}_{\text{tot}}}{dt} = 0$$

$E > p$



① $\vec{F}_1 \parallel \vec{F}_2 \parallel \vec{F}_3$

② Body is in equilibrium

③ $d = 1 \text{ m}$,
 $F_1 = 50 \text{ N}$

④ How big are F_2 and F_3 ?

Forces:

$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

~~$F_1 + F_2 + F_3 = 0$~~

$F_2 - F_3 = -F_1$

Torques:

$\vec{M}_1 + \vec{M}_2 + \vec{M}_3 = 0$

$d \cdot F_1 \sin \phi + 0 = d \cdot F_3 \sin(\pi - \phi) = 0$

$d \cdot F_1 \sin \phi = d \cdot F_3 \sin(\pi - \phi)$

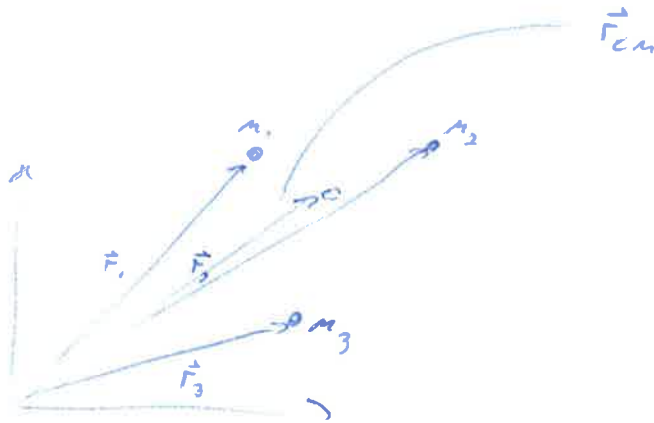
$F_1 = F_3$

Together:

$F_3 = F_1 = 50 \text{ N}$
 $F_2 = -F_1 - F_3 = -2F_1 = -100 \text{ N}$

Center of Mass (Schwerpunkt)

$$\vec{r}_{\text{cm}} = \frac{\sum_{i=1}^N m_i \vec{r}_i}{\sum_{i=1}^N m_i} = \frac{\sum_{i=1}^N m_i \vec{r}_i}{M_{\text{tot}}}$$



$$\vec{r}_s = \frac{\sum_{i=1}^N m_i \vec{r}_i}{\sum_{i=1}^N m_i} = \frac{\sum_{i=1}^N m_i \vec{r}_i}{m_{tot}}$$

$$m_{tot} \vec{r}_s = \sum_{i=1}^N m_i \vec{r}_i$$



$$\frac{d}{dt} (m_{tot} \vec{r}_s) = \sum_{i=1}^N \frac{d}{dt} (m_i \vec{r}_i)$$

$$m_{tot} \frac{d\vec{r}_s}{dt} = \sum_{i=1}^N m_i \frac{d\vec{r}_i}{dt}$$

$$\left. \begin{aligned} \vec{p}_{tot} &= m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 \\ \vec{p}_s &= m_{tot} \vec{v}_s \end{aligned} \right\} \vec{p}_{tot} = \vec{p}_s$$

$$m_{tot} \vec{v}_s = \sum_{i=1}^N m_i \vec{v}_i$$

$$\vec{p}_s = \sum_{i=1}^N \vec{p}_i = \vec{p}_{tot}$$

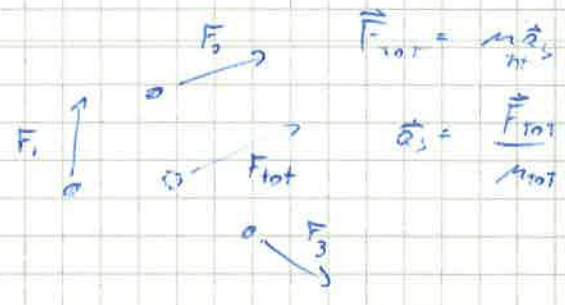
Schwerpunktsimpuls

Gesamtimpuls des System

$$\vec{F}_{tot} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\frac{d}{dt} (m_{tot} \vec{v}_s) = \frac{d}{dt} \sum_{i=1}^N m_i \vec{v}_i$$

$$m_{tot} \frac{d\vec{v}_s}{dt} = \sum_{i=1}^N m_i \frac{d\vec{v}_i}{dt}$$



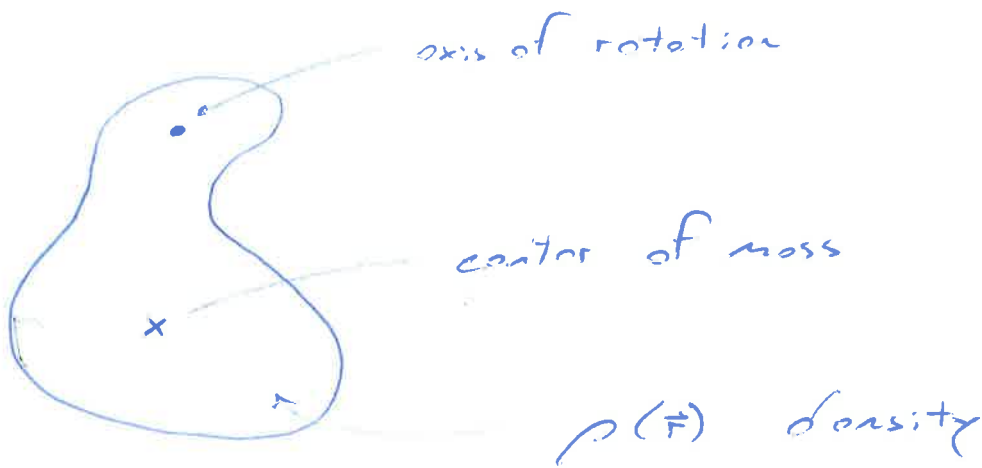
$$m_{tot} \vec{a}_s = \sum_{i=1}^N m_i \vec{a}_i = \sum_{i=1}^N \vec{F}_i = \vec{F}_{tot}$$

Summe der ~~innen~~ aussen einwirkende Kraft.

Wenn die aussen einwirkende Kraft = 0, bewegt sich der Schwerpunkt mit konstanter Geschwindigkeit

Exp!

Physical Pendulum



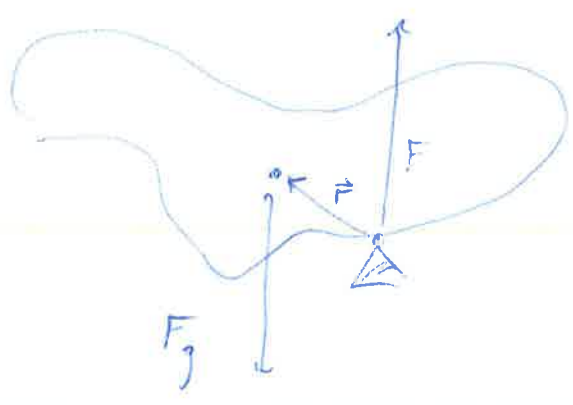
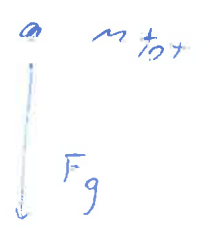
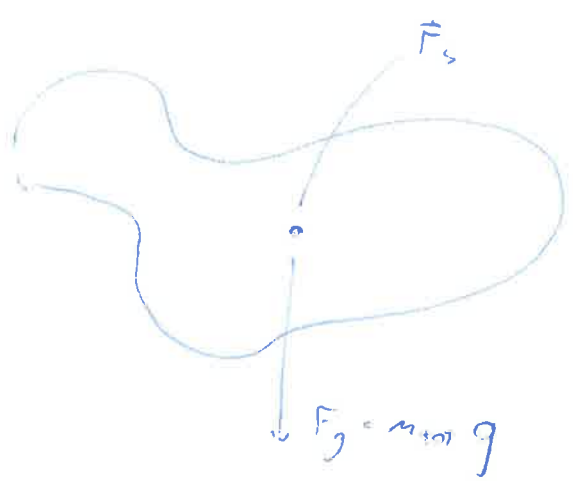
$$\vec{r}_s = \frac{1}{\int \rho(\vec{r}) dV} \int \vec{r} \rho(\vec{r}) dV$$

$$\vec{r}_s = \frac{1}{m} \int \vec{r} \rho(\vec{r}) dV$$

Center of mass swings as if the total mass m were concentrated in it, as in the right drawing.

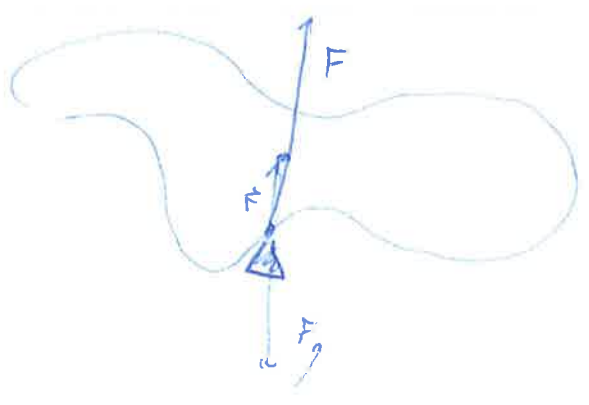
2D

↓ g



Force: $F + F_g = 0$

Torque: $\vec{r} \times \vec{F}_g + 0 \times \vec{F} + 0 \leftarrow$ not in equilibrium

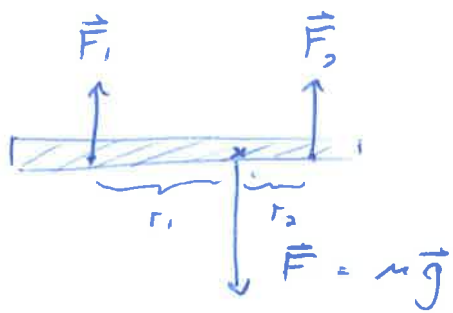


Force: $F + F_g = 0$

Torque: $\vec{r} \times \vec{F}_g + 0 \times \vec{F} = 0$

In equilibrium

23 Exp



Forces: $\vec{F}_1 + \vec{F}_2 - \vec{F}_3 = 0$

$F_1 + F_2 = mg$

Torques: $\vec{M}_1 + \vec{M}_2 = 0$

$-r_1 F_1 + r_2 F_2 = 0$

$r_1 F_1 = r_2 F_2$

∴ For $r_1 > r_2$

$\hookrightarrow F_1 < F_2$

If $F_1 < F_2$ then the friction against finger 1 is smaller than against finger 2. Therefore finger 1 slides more easily. This will bring finger 1 closer to the center of mass until $r_1 \leq r_2$. Then finger 2 will slide more easily... ultimately the two fingers will meet at the center of mass.