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Exercises and Complements for the Introduction to Physics I  
for Students  
of Biology, Pharmacy and Geoscience

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Sheet 1 / September 21, 2017

**Solutions**

**Exercise 1.**

Derivatives  $dy/dx$ :

(a) 
$$\frac{dy(x)}{dx} = 3ax^2 + 2bx + c$$

(b) 
$$\frac{dy(x)}{dx} = \frac{b}{x}$$

(c) 
$$\frac{dy(x)}{dx} = (1 - ax) \exp^{-ax}$$

(d) 
$$\frac{dy(x)}{dx} = \frac{2a - 5abx^3}{2\sqrt{1 - bx^3}}$$

Derivatives with respect to time  $t$ :

(a) 
$$\frac{dE(t)}{dt} = mv(t) \frac{dv}{dt}(t) = mv(t)a(t) = Fv(t) = P$$

(b) 
$$\frac{dp(t)}{dt} = ma(t) = F$$

Integral  $F(x) = \int f(x)dx$ :

(a) 
$$F(x) = \frac{3}{4}x^4 + \frac{2}{3}x^3 + C$$

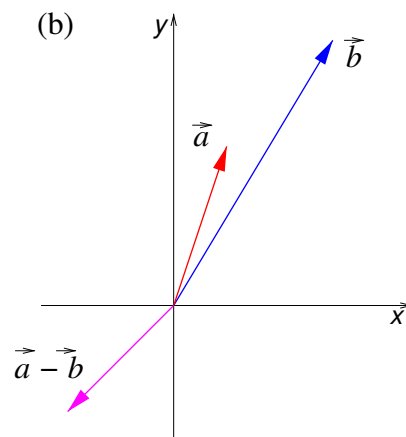
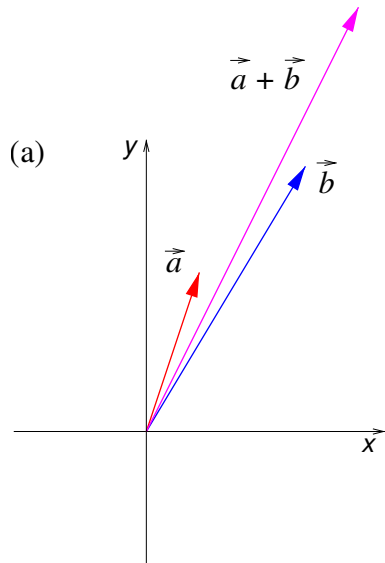
(b) 
$$F(x) = -\frac{a}{b} \cos(bx) + C$$

(c) 
$$F(x) = 4 \ln(x) + C$$

Using the vectors  $\vec{a} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$  and  $\vec{b} = \begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix}$ :

- (a)  $\vec{s} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ -5 \end{pmatrix}$
- (b)  $\vec{s} = \begin{pmatrix} a_1 - b_1 \\ a_2 - b_2 \\ a_3 - b_3 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ -3 \end{pmatrix}$
- (c)  $\vec{d} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} = \begin{pmatrix} 17 \\ -11 \\ -4 \end{pmatrix}$
- (d)  $c = a_1b_1 + a_2b_2 + a_3b_3 = 22$

Graphical solution (2-dimensional):



### Exercise 2.

- (a)  $C_1$  in m and  $C_2$  in m/s  
 (b)  $C_1$  in  $\text{m/s}^2$   
 (c)  $C_1$  in 1/s and  $C_2$  in 1/s  
 (d)  $C_1$  in  $\text{m}^4/(\text{kg} \cdot \text{s}^2)$   
 (e)  $C_1$  in m/s and  $C_2$  in m

### Exercise 3.

General:  $t = \frac{s}{v}$

- (a)  $t = 3.3 \cdot 10^{-24} \text{ s}$   
 (b)  $t = 500 \text{ s} = 8 \text{ min } 20 \text{ s}$   
 (c)  $t = 40000 \text{ s} = 11\text{h } 6 \text{ min } 40\text{s}$

### Exercise 4.

- (a)  $v_{\text{max}} = at_{\text{acceleration}} = 45.4 \text{ km/h}$   
 (b)  $s_{\text{acceleration}} = \frac{1}{2}at_{\text{acceleration}}^2 = 37.8 \text{ m}$

$$(c) s_{constant} = v_{max} t_{constant} = 63.0 \text{ m} \quad s_{braking} = \frac{1}{2} a_{braking} t_{braking}^2 \text{ and } t_{braking} = \frac{v_{max}}{a_{braking}}$$

$$(d) s_{braking} = \frac{v_{max}^2}{2a_{braking}} = 18.9 \text{ m}$$

$$(e) s_{total} = s_{acceleration} + s_{constant} + s_{Brems} = 119.7 \text{ m}$$

### Exercise 5.

(a)

(b)

$$a) v(t_1) > v(t_2) \quad |v(t_1)| > |v(t_2)|$$

$$b) v(t_1) = v(t_2) \quad |v(t_1)| = |v(t_2)|$$

$$c) v(t_1) < v(t_2) \quad |v(t_1)| > |v(t_2)|$$

$$d) v(t_1) > v(t_2) \quad |v(t_1)| = |v(t_2)|$$