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Exercises and Complements for the Introduction to Physics I for Students of Biology, Pharmacy and Geoscience

Sheet 2 / September 26, 2017 **Solutions**

Exercise 6.

The cyclists need an hour till they reach each other, since each has a speed of 16 km/h both ride the bike for 16 km. Accordingly, the bee is flying for one hour back and forth. Since the bee has a velocity of 40 km/h it will fly during this one hour 40 km.

Exercise 7.

The distance of the boat from the landing stage at the time of the jump, $t = 0$, is $e = 10$ m
Length of the boat: $l = 5$ m

We choose the origin of the coordinate system to be where '007' jumps, this is at the edge of the landing stage. Therefore the level of the 'landing place' is y_0 = -3.5 m

First we calculate the time of flight with the equation for vertical motion (free fall):

$$y_0 = -1/2gt_0^2 \Rightarrow t_0 = \sqrt{2y_0/g}$$

The distance s_b which the boat covers during this time is given by

$$s_b = v \cdot t_0$$

Thus the minimal horizontal flight path is:

$$s_{min} = e + v \cdot t_0$$

In this case, the motorbike is landing with the center just on the stern(end) of the boat. The maximal horizontal flight path is given by:

$$s_{max} = e + l + v \cdot t_0$$

In this case, the motorbike is landing with the center on the bow (forward part) of the boat. Resulting for the constant horizontal velocity:

$$v_{min} = s_{min}/t_0 = 73 \text{ km/h} \qquad \text{and} \qquad v_{max} = s_{max}/t_0 = 94 \text{ km/h}$$

The speed of James Bond has to be between 73 km/h and 94 km/h.

Exercise 8.

General:

$$v = \dot{s} = \frac{ds}{dt} \qquad \text{and} \qquad a = \ddot{s} = \frac{d^2s}{dt^2}$$

(a) The derivative of a vector has to be calculated for each component:

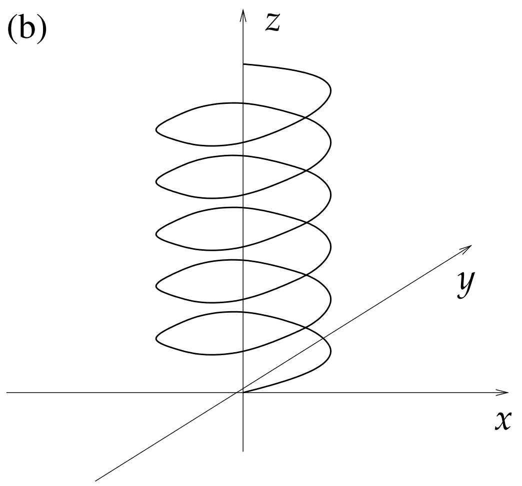
$$\begin{aligned} \dot{x}(t) &= v_x(t) = -r\omega \cdot \sin(\omega t) & \ddot{x}(t) &= a_x(t) = -r\omega^2 \cdot \cos(\omega t) \\ \dot{y}(t) &= v_y(t) = r\omega \cdot \cos(\omega t) & \ddot{y}(t) &= a_y(t) = -r\omega^2 \cdot \sin(\omega t) \\ \dot{z}(t) &= v_z(t) = v_z & \ddot{z}(t) &= a_z(t) = 0 \end{aligned}$$

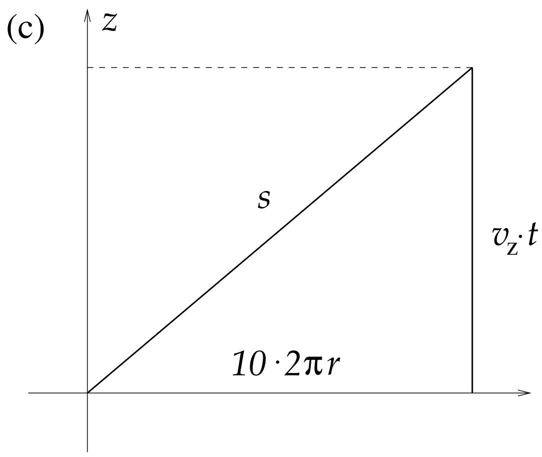
and the absolute values are:

$$\begin{aligned} |v(t)| &= \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{r^2\omega^2 \sin^2(\omega t) + r^2\omega^2 \cos^2(\omega t) + v_z^2} \\ &= \sqrt{r^2\omega^2 \underbrace{(\sin^2(\omega t) + \cos^2(\omega t))}_{=1} + v_z^2} = \sqrt{r^2\omega^2 + v_z^2} = 6.28 \text{ m/s} \end{aligned}$$

$$\begin{aligned} |a(t)| &= \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{r^2\omega^4 \cos^2(\omega t) + r^2\omega^4 \sin^2(\omega t) + 0} \\ &= \sqrt{r^2\omega^4 \underbrace{(\sin^2(\omega t) + \cos^2(\omega t))}_{=1}} = \sqrt{r^2\omega^4} = 39.5 \text{ m/s}^2 \end{aligned}$$

(b) The point describes a spiral around the z-axis, see figure.





(c) It is necessary to think about the number of turns which the point makes in order to calculate the path. Since the angular frequency is $\omega = 2\pi$, the point passes one winding per second. Hence, in 10 s it passes 10 windings, and each has a circumference of $2\pi r$. But the point is also moving in z-direction which also has to be considered. Resulting in a right-angle triangle shown in the figure (c). The path can now be calculated by using the Theorem of Pythagoras:

$$s = \sqrt{(10 \cdot 2\pi r)^2 + v_z^2 t^2} = 62.9 \text{ m}$$

Exercise 9.

(a) Centripetal acceleration($a_r = a_c$): $a_c = v_1^2/r$. The centripetal acceleration has the biggest value at the point where the vehicle is exiting the curve, due to that the final velocity is:

$$v_1 = \sqrt{a_c r} = 13.89 \text{ m/s} = 50 \text{ km/h}$$

(b) The total acceleration $\vec{a}$ is composed of the centripetal $\vec{a_c}$ and the tangential $\vec{a_t}$ acceleration, see

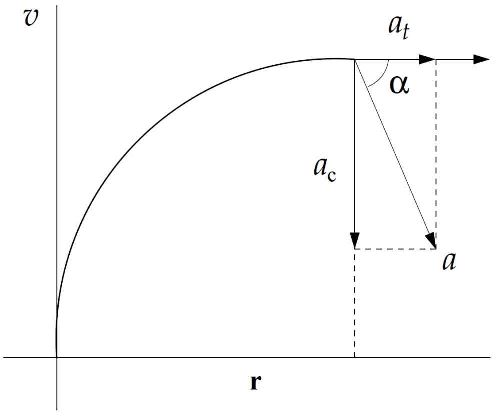


figure. So it is necessary to determine the tangential acceleration. Since the acceleration has the biggest value at the exit of the curve only this point needs to be investigated. The covered distance s_1 is:

$$s_1 = \frac{1}{2}a_t t^2 + v_0 t$$

The velocity v_1 is:

$$v_1 = a_t t + v_0$$

solve the equation for t and substitute t into s_1:

$$t = \frac{v_1 - v_0}{a_t}$$

$$s_1 = \frac{1}{2}a_t \frac{v_1^2 - 2v_1 v_0 + v_0^2}{a_t^2} + \frac{v_0 v_1 - v_0^2}{a_t} = \frac{v_1^2 - v_0^2}{2a_t}$$

out of this and by knowing the tangential acceleration it is possible to calculate the speed:

$$v_1 = \sqrt{v_0^2 + 2a_t s_1}$$

or the tangential acceleration, by using $s_1 = \frac{1}{4}2\pi r$:

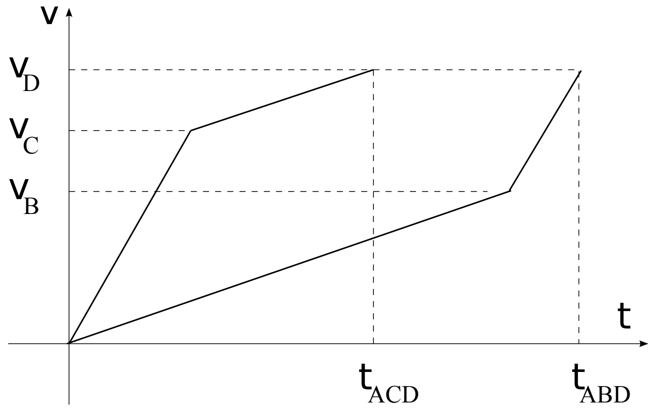
$$a_t = \frac{v_1^2 - v_0^2}{2s_1} = \frac{v_1^2 - v_0^2}{\pi r} = 0.787 \ \frac{\text{m}}{\text{s}^2}$$

The absolute value of the total acceleration $\vec{a}$ therefor is:

$$a = \sqrt{a_c^2 + a_t^2} = 3.94 \text{ m/s}^2 \qquad \tan\alpha = \frac{a_c}{a_t} \quad \Rightarrow \quad \alpha = 78.5°$$

Additional Exercise.

The following considerations are only true if no friction is present. The velocities at D are equal, since the distance and the experienced acceleration is the same for both paths. In the path 1 (ABD): the sphere accelerates from A to B slowly since the inclination is small. In the $v(t)$ plot it is represented by a line with a small slope and accordingly a long time passes till the sphere reaches B. By a free fall the sphere reaches D starting from B, so it gets accelerated from a low velocity to a high. Correspondingly the slope of the line is steep in the $v(t)$ - diagram.



Path 2 (ACD): the free fall occurs first, so the sphere gets rapidly accelerated and reaches C at a high velocity. The acceleration from C to D is small but due to the high velocity the distance is covered fast.