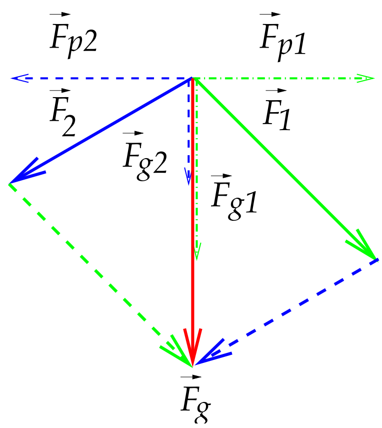
Departement Physik
Universität Basel
Prof. M. Poggio / PD Dr. M. Calame
T. Meier / C. Drechsel
tobias.meier@unibas.ch
c.drechsel@unibas.ch
Büro 3.04
Tel.: 061 207 37 30
http://adam.unibas.ch

Exercises and Complements for the Introduction to Physics I for Students of Biology, Pharmacy and Geoscience

Sheet 3 / September 26, 2017 **Solutions**

Exercise 10.



Each force F_1 and F_2 can be split in a force component F_p which acts parallel to the attachment and perpendicular to it F_g, respectively, i.e. downwards (see figure). The absolute value of the two parallel components F_{p1} and F_{p2} has to be equal (they just differ in the sign, since they act in the opposite direction), since the system is in equilibrium.

$$\begin{aligned} F_{p1} &= F_{p2} \\ F_1 \sin(\alpha) &= F_2 \sin(\beta) \\ F_1 &= F_2 \frac{\sin(\beta)}{\sin(\alpha)} \end{aligned}$$

The sum of the perpendicular components is equal to the total force which is pulling the mass downwards, the gravitational force:

$$F_{g1} + F_{g2} = F = mg$$

the perpendicular components are:

$$F_{g1} = F_1 \cos(\alpha) \qquad F_{g2} = F_2 \cos(\beta)$$

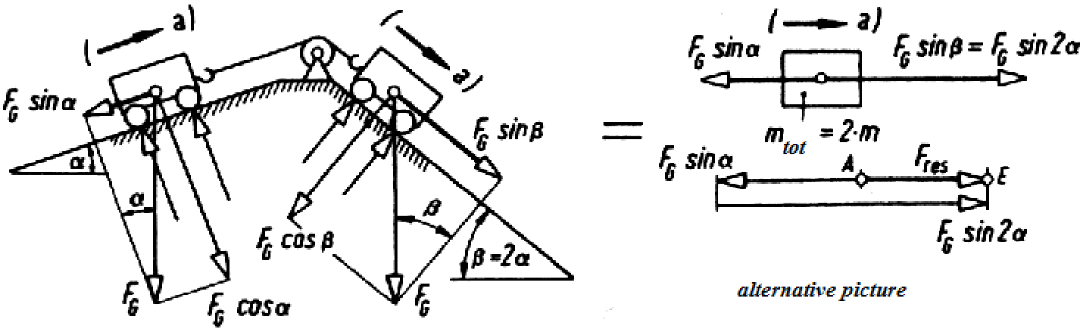
From this it follows:

$$\begin{aligned} F = mg &= F_1 \cos(\alpha) + F_2 \cos(\beta) \\ &= F_2 \frac{\sin(\beta)}{\sin(\alpha)} \cos(\alpha) + F_2 \cos(\beta) \\ &= F_2 \frac{\cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)}{\sin(\alpha)} \end{aligned}$$

Solve for F_2:

$$\begin{aligned} F_2 &= F \frac{\sin(\alpha)}{\cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)} = 3.58\text{ N} \\ F_1 &= F \frac{\sin(\beta)}{\cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)} = 4.39\text{ N} \end{aligned}$$

Exercise 11.



a) Only the components of F_G which act parallel to the surface have to be considered for the calculation of the total force F_{tot}:

$$F_{tot} = -F_G \sin\alpha + F_G \sin 2\alpha = m_{tot} a = 2ma$$

Use $F_G = mg$ and solve for a:

$$a = \frac{g(\sin 2\alpha - \sin\alpha)}{2} = 1.476 \quad \mathrm{m/s^2}$$

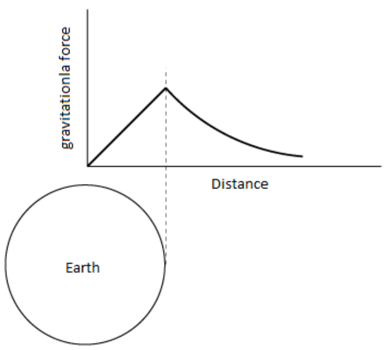
The acceleration a is then independent of the mass of the wagons.

b) For $\Delta t = 10$ s:

$$v = a\Delta t = 14.76 \quad \mathrm{m/s}$$

Exercise 12.

The correct answer is (b). The gravitational force inside the cavity is smaller, because a part of the Earth's mass is above and therefore this mass is pulling up. This effect is partially compensating the effect of the mass which is below the feet. The gravitational force acting on things is biggest on the Earth's surface.



Exercise 13.

(a) The geostationary orbit has to rotate with the same angular velocity around the Earth as the Earth itself is rotating.

$$\omega = \frac{2\pi}{86400} = 7.27 \cdot 10^{-5} \quad \frac{1}{\mathrm{s}}$$

The satellite just stays on a circular path if the centrifugal force F_C (script 103-7) is equal the gravitational force F_G (script 103-5):

$$F_C = F_G$$
$$m\omega^2 r = \gamma \frac{mM}{r^2}$$
$$\omega^2 r = \gamma \frac{M}{r^2}$$

m	mass of the satellite
M	mass of the Earth
γ	constant of gravitation
r	distance to the center of the Earth

from this it follows that the distance to the Earth's center is:

$$\begin{aligned} r &= \sqrt[3]{\frac{\gamma M}{\omega^2}} \\ &= 42300 \quad \text{km} \end{aligned}$$

and the distance to the Earth's surface is therefore 36000 km.

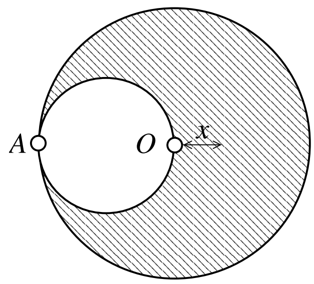
(b) The orbital plane of the satellite has to go through the Earth's center. In the cross sectional view with the Earth's surface, it is a great circle. The satellite is really geostationary only if the great circle is at the equator. Otherwise, the satellite would oscillate with a period of one day between northern and southern hemisphere.

Additional Exercise (the additional exercise is not relevant to the exam. It is for the students which are looking for a challenge.)

The total area is πR^2 and the area of the opening is $\pi\left(\frac{R}{2}\right)^2$. As a result, the area for the shaded region is:

$$\pi R^2 - \pi(\frac{R}{2})^2 = \frac{3}{4}\pi R^2$$

In order to calculate the position of the center of mass, through which the rotation axis goes, we picture the circular disk with a hole as an overlap of a massive disk (radius R) and a round disk with a negative mass (radius $R/2$). The mass is proportional to the area of the disk. Since the mass of the shadowed region is m then the mass of the uniform disk with the radius R is $\frac{4}{3}m$ and the mass of the hole is $-\frac{1}{3}m$. x is the distance from the center of the disk to the point where the rotation axis goes through, see figure. Then, according to the center-of-mass theorem the following holds:



$$x = \frac{\frac{4}{3}m \cdot 0 + (-\frac{1}{3}m)(-\frac{R}{2})}{\frac{4}{3}m - \frac{1}{3}m} = \frac{R}{6}$$

thereby we use positive values for the distances away (right side) from the hole. In general the moment of inertia of a disk is given by:

$$J_{disk} = \frac{1}{2}mr^2$$

Using Steiner's Theorem (parallel axis Theorem) we obtain the moment of inertia in relation to the point x:

$$\begin{aligned} J_{massive\ disk} &= \left(\frac{1}{2}(\frac{4}{3}m)R^2 + \frac{4}{3}m\left(\frac{R}{6}\right)^2\right) \\ J_{hole} &= \left(\frac{1}{2}(-\frac{1}{3}m)\left(\frac{1}{2}R\right)^2 - \frac{1}{3}m\left(\frac{R}{2}+\frac{R}{6}\right)^2\right) \end{aligned}$$

It is allowed to sum up the moments of inertia if the axis of the center-of-mass is the same, and we obtain:

$$\begin{aligned} J &= J_{massive\ disk} + J_{hole} \\ J &= \left(\frac{2}{3}mR^2 + \frac{1}{27}mR^2\right) - \left(\frac{1}{24}mR^2 + \frac{4}{27}mR^2\right) \\ J &= \frac{37}{72}\, mR^2 \;=\; 0.15\ kgm^2 \end{aligned}$$