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Exercises and Complements for the Introduction to Physics I for Students of Biology, Pharmacy and Geoscience

Sheet 6 / October 13, 2017 **Solutions**

Exercise 22. (a) In this case the equations for an elastic collision are valid, according to them the velocity is given by: $v_1' = -v_2'$. The negative sign indicates that the objects are moving in opposite directions. From the equations (4-5) and (4-6) in Trautwein page 39 it results for v_1' and v_2':

$$\frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} = \frac{(m_2 - m_1)v_2 + 2m_1v_1}{m_1 + m_2}$$

$$\frac{(m_1 - m_2)v_1 + 0}{m_1 + m_2} = \frac{0 + 2m_1v_1}{m_1 + m_2}$$

$$\frac{(m_1 - m_2)v_1}{m_1 + m_2} = \frac{2m_1v_1}{m_1 + m_2}$$

$$(m_1 + m_2)v_1 = -2m_1v_1$$

$$m_1v_1 - m_2v_1 = -2m_1v_1$$

$$-m_2v_1 = -3m_1v_1$$

$$m_2 = 3m_1$$

$$\Rightarrow m_2 = 6\,kg$$

(b) According to the previous equation it follows:

$$v_1' = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2}$$

$$\Rightarrow v_1' = -3.35\,m/s$$

Since $v_1' = -v_2'$ the velocity of the second object is $v_2' = 3.35\,m/s$. It is also possible to calculate v_2' directly from the equation for v_2' mentioned in (a). The absolute value of the velocity for both objects is $3.35\,m/s$.

Exercise 23. The rolling wagon has a momentum in horizontal direction. The rain falls perpendicular to the motion of the wagon into it, therefore it has no horizontal component of the momentum which can be transmitted into the momentum of the wagon. As a result, the momentum of the wagon does not change. The mass of the wagon is increasing by the weight of the water which falls into it. The mass is increasing and the momentum is constant therefore the velocity has to decrease and due to it also the kinetic energy.

m is the mass of the collected water. The outflowing water is reducing the mass with the rate of dm/dt. Due to that, the momentum is reduced by the rate of $dp/dt = dm/dt \cdot v$. As a result the momentum of the wagon is $dP/dt = dM/dt \cdot v + M \cdot dv/dt$, where M is the total mass of the wagon. Based on the conservation of momentum is $dP/dt = dp/dt$ and because of the law of conservation of mass is $dM/dt = dm/dt$. From this it follows that $dv/dt = 0$ and so the wagon continues to drive with a constant velocity. The outflowing water exerts force on the wagon, but this gets compensated by the reduction of the mass.

Exercise 24. The common velocity v' of the vehicles after the crash (inelastic collision) results from the law of conservation of momentum:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)v' \quad \text{then} \quad v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

The energy which gets transformed into heat in this process is:

$$\Delta E = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} - \frac{(m_1 + m_2)v'^2}{2}$$

(a) Given in the problem was that: $m_1 = m_2 = m$, $v_1 = v$ and $v_2 = -v$. From this it follows $v' = 0$ and $\Delta E = mv^2$, i.e. the original available energy of the vehicles $E_{kin} = 2 \cdot (mv^2/2)$ is completely used for the deformation of the vehicles.

(b) In this case it was given that $v_1 = 2v$ and $v_2 = 0$. Under these conditions the original available kinetic energy is $(m/2)(2v)^2 = 2mv^2$, twice as much as in (A). From this, it follows that $v' = v$ and $\Delta E = mv^2$. Accordingly the same amount of the kinetic energy is used for the deformation as in (a). Since the initial energy was higher, after the collision each vehicle has a kinetic energy of $mv^2/2$.

Exercise 25. Due to the conservation of angular momentum it is necessary that the momenta for the outstreched arms $L_0 = J_0\omega_0$ and with the arms closer to the body $L_1 = J_1\omega_1$ have to be equal, $L_0 = L_1$.
By solving this we obtain $\omega_1 = \omega_0 \cdot \frac{J_0}{J_1}$.
For the moments of inertia we calculate:

$$J_0 = J_P + J_C + 2mr_0^2 = 1.95kg \cdot m^2 + 0.27kg \cdot m^2 + 2 \cdot 2kg \cdot 0.75^2 \tag{1}$$

and

$$J_1 = J_P + J_C + 2mr_1^2 = 1.95kg \cdot m^2 + 0.27kg \cdot m^2 + 2 \cdot 2kg \cdot 0.1^2 \tag{2}$$

and with $\omega_0 = 1\,\frac{\pi}{s}$ we obtain $\omega_1 \approx 2\,\frac{\pi}{s}$.

Exercise 26.
(a) The components of the force in z-direction and perpendicular to it in x, y-direction are:

$$F_z = F\cos\psi \qquad \text{and} \qquad F_\varphi = F\sin\psi$$

the one in x- and y-direction are:

$$F_x = F_\varphi \cos\varphi = F\sin\psi\cos\varphi \qquad \text{and} \qquad F_y = F_\varphi \sin\varphi = F\sin\psi\sin\varphi$$

Therefore the normal stress perpendicular to the x, y-plane is:

$$\sigma_{zz} = \frac{F_z}{A} = \frac{F}{A}\cos\psi = 70.7 \quad \text{MPa}$$

The shear stress in the x, y-plane in x- respectively in y-direction are:

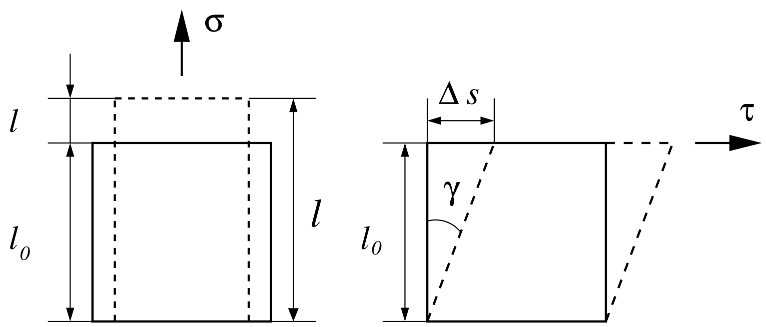
$$\begin{aligned}\tau_{zx} &= \frac{F_x}{A} = \frac{F}{A}\sin\psi\cos\varphi = 61.2 \quad \text{MPa} \\ \tau_{zy} &= \frac{F_y}{A} = \frac{F}{A}\sin\psi\sin\varphi = 35.4 \quad \text{MPa}\end{aligned}$$

(b) The normal stress σ (tension or compression stress) creates expansions respectively compression (relative change in length).

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0}$$

The shear stress τ (tangential stress) creates shear (shear angle):

$$\gamma \approx \tan\gamma = \frac{\Delta s}{l_0}$$



Hooke's Law can be used for linear elastic materials:

$$\sigma = E\varepsilon \qquad \text{bzw.} \qquad \tau = G\gamma$$

where E is the elastic modulus and G the shear modulus. Therefore we obtain for the aluminum cube:

$$\begin{aligned}\varepsilon_{zz} = \sigma_{zz}/E = 9.7 \cdot 10^{-4} \text{ } (\approx 0.1\% \text{ expansion}) \quad &\Rightarrow \quad \Delta l = \varepsilon_{zz} l_0 \approx 0.01 \quad \text{mm} \\ \gamma_{zx} = \tau_{zx}/G = 2.3 \cdot 10^{-3} \text{ } (\approx 0.23\% \text{ shear}) \quad &\Rightarrow \quad \Delta s_x = \gamma_{zx} l_0 \approx 0.023 \quad \text{mm} \\ \gamma_{zy} = \tau_{zy}/G = 1.3 \cdot 10^{-3} \text{ } (\approx 0.13\% \text{ shear}) \quad &\Rightarrow \quad \Delta s_y = \gamma_{zy} l_0 \approx 0.013 \quad \text{mm}\end{aligned}$$