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Exercises and Complements for the Introduction to Physics I for Students of Biology, Pharmacy and Geoscience

Sheet 7 / October 13, 2017 **Solutions**

Exercise 27.
Since the two pistons are at the same height, the pressure of the liquid at both pistons (when the system is in equilibrium) is the same:

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

with

$$F_2 = (m + m_K)g \qquad \text{follows} \qquad F_1 = (m + m_K)g\frac{A_1}{A_2} = 87.2 \quad \text{N}$$

Exercise 28. From the capillary law (script 107-9) it follows that:

$$\sigma_{1,3} - \sigma_{1,2} = \sigma_{2,3} \cos\theta$$

where $\sigma_{2,3} = \sigma$ is the surface tension of water towards air/vapor. Using this result and substituting it in the formula for calculating the height of the liquid column it follows:

$$r = \frac{2(\sigma_{1,3} - \sigma_{1,2})}{hg\rho} = \frac{2\sigma\cos\theta}{hg\rho} = 1.13 \quad \mu\text{m}$$

$$d = 2r = 2.26 \quad \mu\text{m}$$

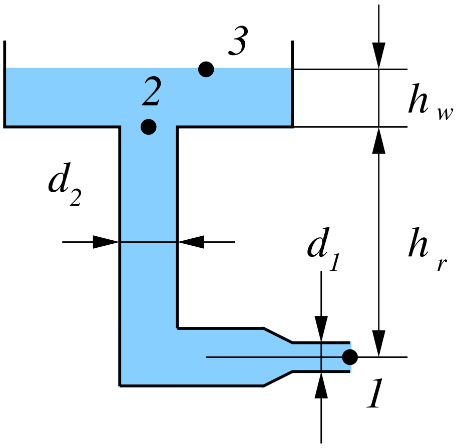
Exercise 29.
(a) In general, according to Bernoulli:

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_3 + \rho g h_3 + \frac{1}{2}\rho v_3^2 = \text{const}$$

For this exercise: $p_1 = p_3 =$ the pressure of air, $h_1 = 0$, $h_3 = h_r + h_w$, ρ density of water , and $v_3 = 0$ (velocity at point 3, see figure) since the level of the water is constant.

$$\frac{1}{2}\rho v_1^2 = \rho g(h_r + h_w) \qquad \Rightarrow \qquad v_1 = \sqrt{2g(h_r + h_w)} = 16.57 \quad \text{m/s}$$

(b) Due to the equation of continuity, it follows:



$$v_2 A_2 = v_1 A_1 \qquad \text{with} \qquad A_i = \pi \left(\frac{d_i}{2}\right)^2$$

where A_i is the cross section at the corresponding position. From this it follows:

$$v_2 d_2^2 = v_1 d_1^2 \qquad \Rightarrow \qquad v_2 = 7.36 \quad \text{m/s}$$

(c) Using again the Bernoulli equation:

$$p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 = p_3 + \rho g h_3 + \frac{1}{2}\rho v_3^2 = \text{const}$$

Here: $p_3 = p_0 =$ pressure of air, $h_2 = h_r$, $h_3 = h_r + h_w$, v_2 calculated in (b), $v_3 = 0$.

$$p_2 = p_0 + \rho g h_w - \frac{1}{2}\rho v_2^2 = 1.11 \quad \text{bar}$$

Exercise 30.
(a) The flow of water through a cylindrical tube is described by:

$$R_0 = \frac{8}{\pi}\frac{\eta L}{r_0^4}$$

The total resistance of the new tube is equal to the resistance of the four parallel tubes:

$$\frac{1}{R_n} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{4}{R_0}$$

Therefore is:

$$R_n = \frac{1}{4}R_0$$

$$\frac{8\eta L}{\pi r_n^4} \stackrel{!}{=} \frac{1}{4}\frac{8}{\pi}\frac{\eta L}{r_0^4}$$

$$\Rightarrow r_n = \sqrt[4]{4}\, r_0 = \sqrt{2}\, r_0 = 0.141 \text{ m}$$

(b) Reynolds number:

$$I_0 = I_1$$

$$4 v_0 A_0 \stackrel{!}{=} v_1 A_1$$

$$\Rightarrow v_0 = \frac{1}{2} v_1$$

Reynolds number: $Re = \dfrac{\rho v d}{\eta}$

$$\frac{Re_0}{Re_1} = \frac{v_0 r_0}{v_1 r_1}$$

$$\frac{Re_0}{Re_1} = \frac{v_0 r_0}{2 v_0 \sqrt{2} r_0}$$

$$\frac{Re_0}{Re_1} = \frac{1}{2\sqrt{2}}$$

(c) In the case of A_1 a turbulent flow is more probable. The velocity v_1 is higher and therefore the Reynolds number Re_1 is closer to the critical Reynolds number where turbulent flow occurs.

Additional exercise:

(a) The forces can be described by the following equations:

$$\begin{aligned} mg &= V_K \rho_K g \\ F_A &= V_W \rho_W g \end{aligned}$$

where V_K is the volume of the cuboid and ρ_K the density, V_W is the volume of the cuboid which is in the water and ρ_W is the density of water. From this it follows:

$$V_K \rho_K = V_W \rho_W \quad \Rightarrow \quad \frac{\rho_K}{\rho_W} = \frac{V_W}{V_K} = \frac{90}{100} \quad \Rightarrow \quad \rho_K = \frac{90}{100}\rho_W$$

(b) Due to the additional buoyancy of the oil, the volume of the cuboid which enters the water is smaller.

(c)

$$(h_O + h_W) A \rho_K g = h_W \rho_W A g + h_O \rho_O A g$$

$$(h_O + h_W)\rho_K = h_W \rho_W + h_O \rho_O$$

$$\frac{h_W}{h_O} = \frac{\rho_O - \rho_K}{\rho_K - \rho_W} = \frac{1}{2}$$

1/3 of the volume is in the water, since the height is divided at the rate of 1:2.