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Exercises and Complements for the Introduction to Physics I for Students of Biology, Pharmacy and Geoscience

Sheet 9 / November 7, 2017 **Solutions**

Exercise 36. The amplitude of a forced oscillation is (Script 108-8):

$$a_0(\omega) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2}}$$

where δ is the damping. Even a small force F_0 gets the system to swing if the excitation frequency ω is close to the eigenfrequeny (resonance frequency) ω_0 of the system (and if the excitation has the right phase shift). That is relatively easy to figure out for a church bell, since the frequency of the oscillation (not of the sound) is rather small and intuitive to find as well as the proper phase shift.

Exercise 37. After the clock is running for 12 hours it shows 11.5 h, so it made just $11.5/12 = 95{,}8\%$ of the required pendulum motions, respectively the time of oscillation T_0 is too big. In order for the clock to be on time:

$$T = T_0 \cdot 0.958$$

The equation for a mathematical pendulum (pendulum clock) describing the time of oscillation is:

$$T_0 = 2\pi\sqrt{\frac{l_0}{g}}$$

and from this it follows:

$$\frac{T'}{T_0} = \sqrt{\frac{l'}{l_0}} = 0.958 \qquad \Rightarrow \qquad l' = l_0 \cdot 0.958^2 = 0.459 \text{ m}$$

Exercise 38.
(a) The eigenfrequency of a mathematical pendulum is:

$$\omega_P = 2\pi/T_P = \sqrt{g/l}$$

The eigenfrequency of the combined system of spring and pendulum is:

$$\omega_F = 2\pi/T_F = \sqrt{D/m + g/l}$$

for the time of contact t it must be $t = T_F/2$ (since the spring is considered as massless) and for T_F:

$$T_F = \frac{2\pi}{\sqrt{D/m + g/l}}$$

and consequently t is:

$$t = T_F/2 = \frac{1}{2} \cdot \frac{2\pi}{\sqrt{D/m + g/l}} = 0.32\ s$$

(b) Since the time of the oscillation of the pendulum (for small deflections) is independent of the angle and $T_P = T_F \cdot \sqrt{2}$, is the time of contact independent of α.

Exercise 39.

(a) The following force is needed to bring the cuboid out of equilibrium by pushing it by the distance Δh into the water:

$$F = m_{water} g = V_{water} \rho_{water} g = A \Delta h \rho_{water} g$$

whereby the force is proportional to the deflection Δh (compare with the behavior of a spring) and the systems is oscillating harmonically.

(b) the constant of proportionality (spring constant):

$$c = \frac{F}{\Delta h} = A \rho_{water} g$$

The following equation describes the harmonic oscillation:

$$T = 2\pi \sqrt{\frac{m_{cuboid}}{c}} = 2\pi \sqrt{\frac{A h \rho_{cuboid}}{A \rho_{water} g}} = 2\pi \sqrt{\frac{h \rho_{cuboid}}{\rho_{water} g}}$$

(c) In the case of a wooden sphere (instead of the cuboid) the cross-sectional area is not constant in height. As a result, the buoyant force is not proportional to the depth of immersion (deflection) and as a consequence the sphere is not oscillating harmonically. Therefore the result obtained in (b) is not valid for a sphere.

Additional Exercise

(a) It is given that (script 108-6):

$$x(t) = c_0 e^{-\delta t} \sin(\omega t - \phi_0)$$

accordingly is:

$$x(t_0) = c_0 e^{-\delta t_0} \sin(\omega t_0)$$
$$x(t_0 + 5T) = c_0 e^{-\delta(t_0+5T)} \sin(\omega(t_0 + 5T)) = c_0 e^{-\delta(t_0+5T)} \sin(\omega t_0)$$

The ratio is:

$$\frac{x(t_0 + 5T)}{x(t_0)} = \frac{1}{2} = e^{-5\delta T}$$

and therefore

$$\delta = \frac{ln 2}{5T} = 0.0462\ \mathrm{s}^{-1}$$

(b) At the maximum deflection the velocity of the brick is zero. In this position the brick is standing still and in order to move again it is necessary to overcome the static friction. The deflection gets smaller and smaller and at a certain point the force of the static friction is bigger than the force of the spring. The result is, that the brick is not moving anymore.

(c) Soap reduces the damping δ. Since $\omega^2 = \omega_0^2 - \delta^2$, where $\omega = \frac{2\pi}{T}$ which gets bigger and therefore the oscillation time gets reduced.